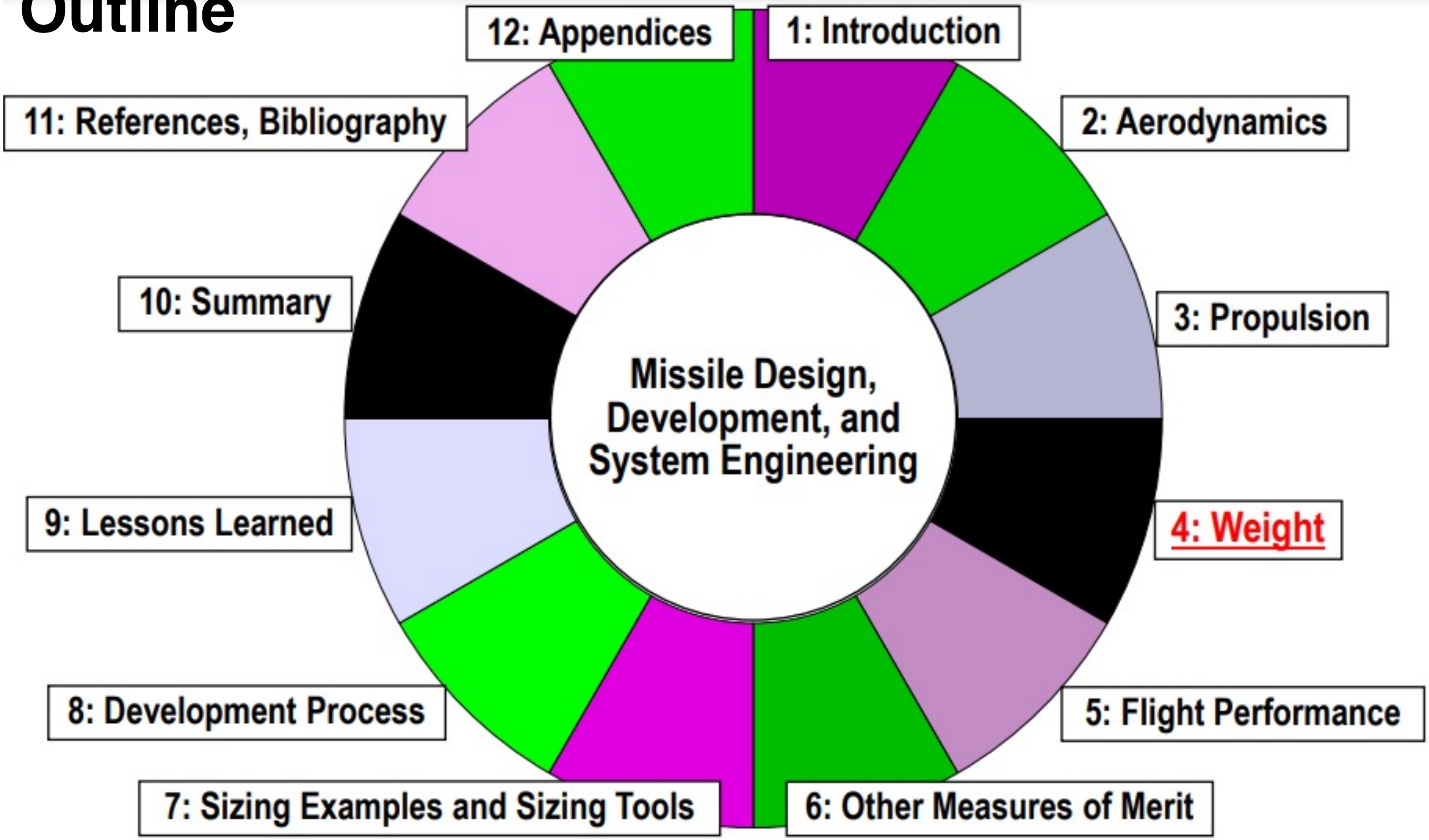
Chapter 4: Weight Considerations in Missile Design, Development, and System Engineering

Outline



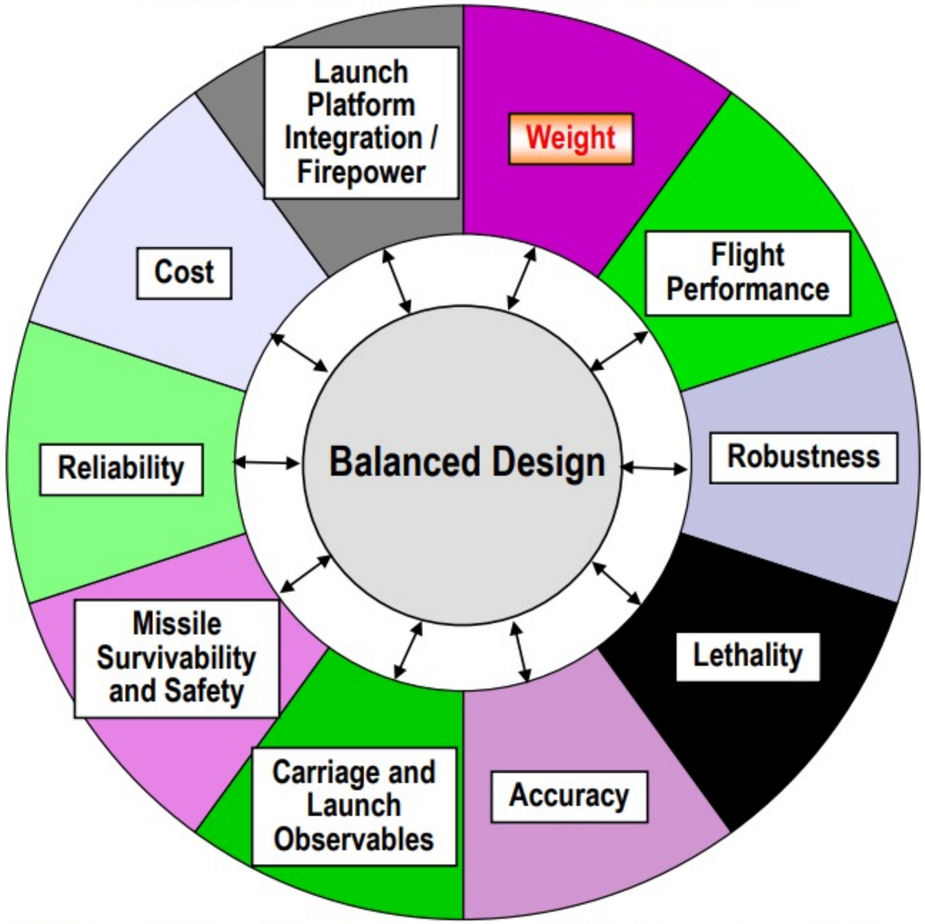
Chapter 4: Weight Considerations in Missile Design, Development, and System Engineering

What You Will Learn

- Conceptual Design Methods for Predicting Missile Weight
- Missile Airframe Manufacturing Processes
- Design and System Engineering Tradeoffs That Impact Missile Weight
- New Technologies for Missile Weight Reduction

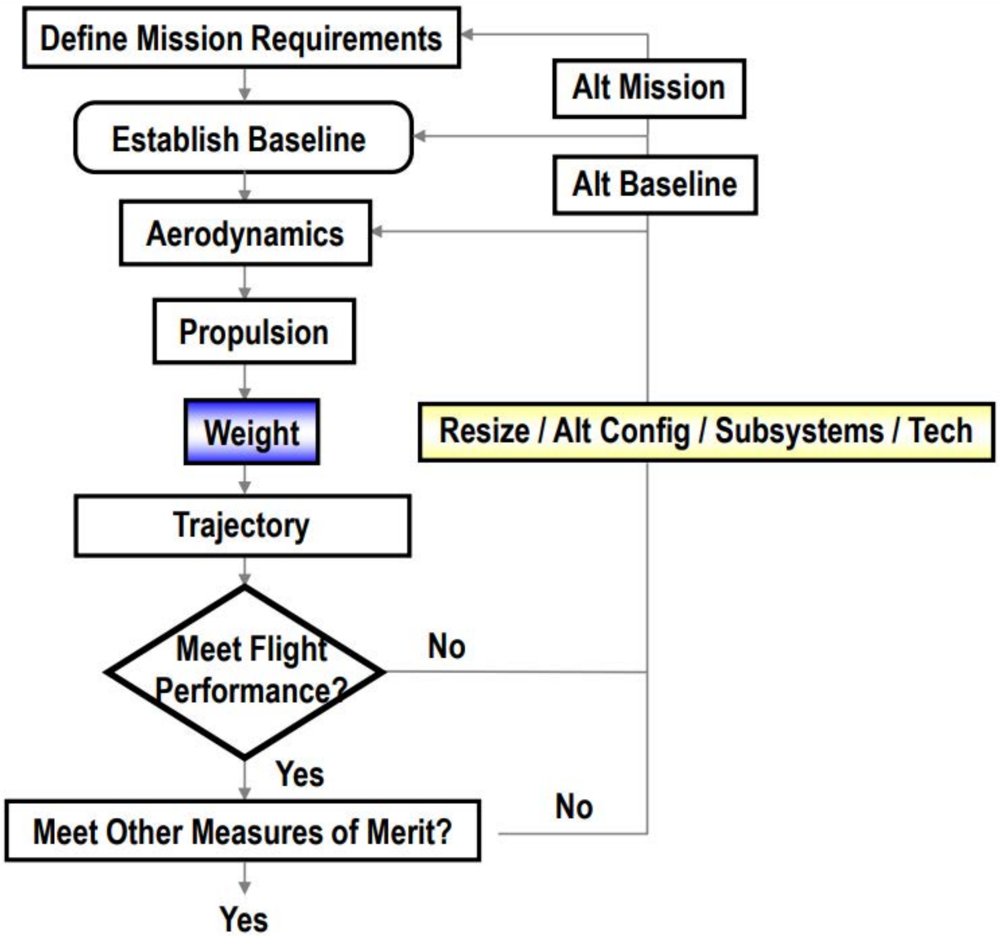
A Balanced Missile Design Requires Harmonized Mission Requirements and Measures of Merit

Example of Alternative Measures of Merit

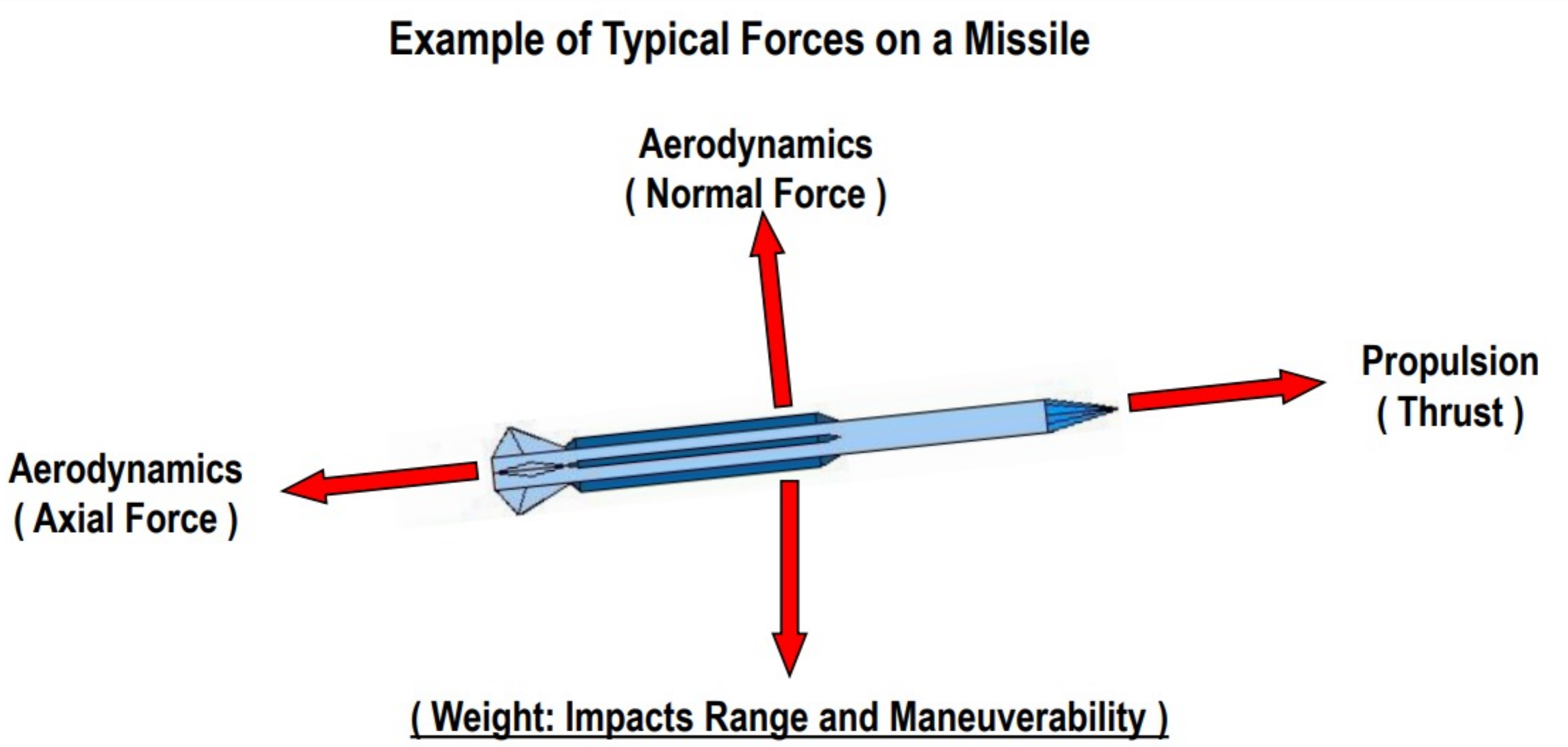


Note: House of Quality, Pareto sensitivity, and Design of Experiments (DOE) may be used in harmonizing measures of merit

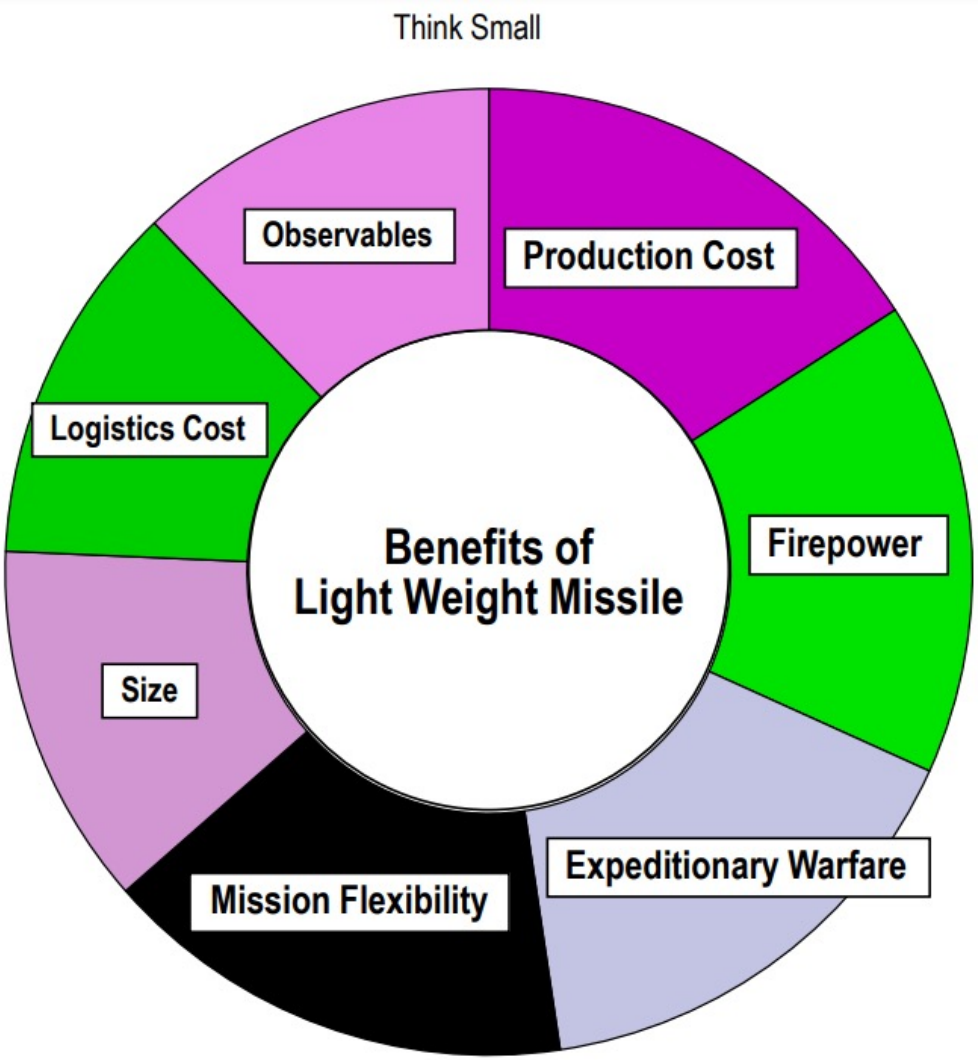
Conceptual Design and System Engineering Require Broad, Creative, Rapid, and Iterative Evaluations



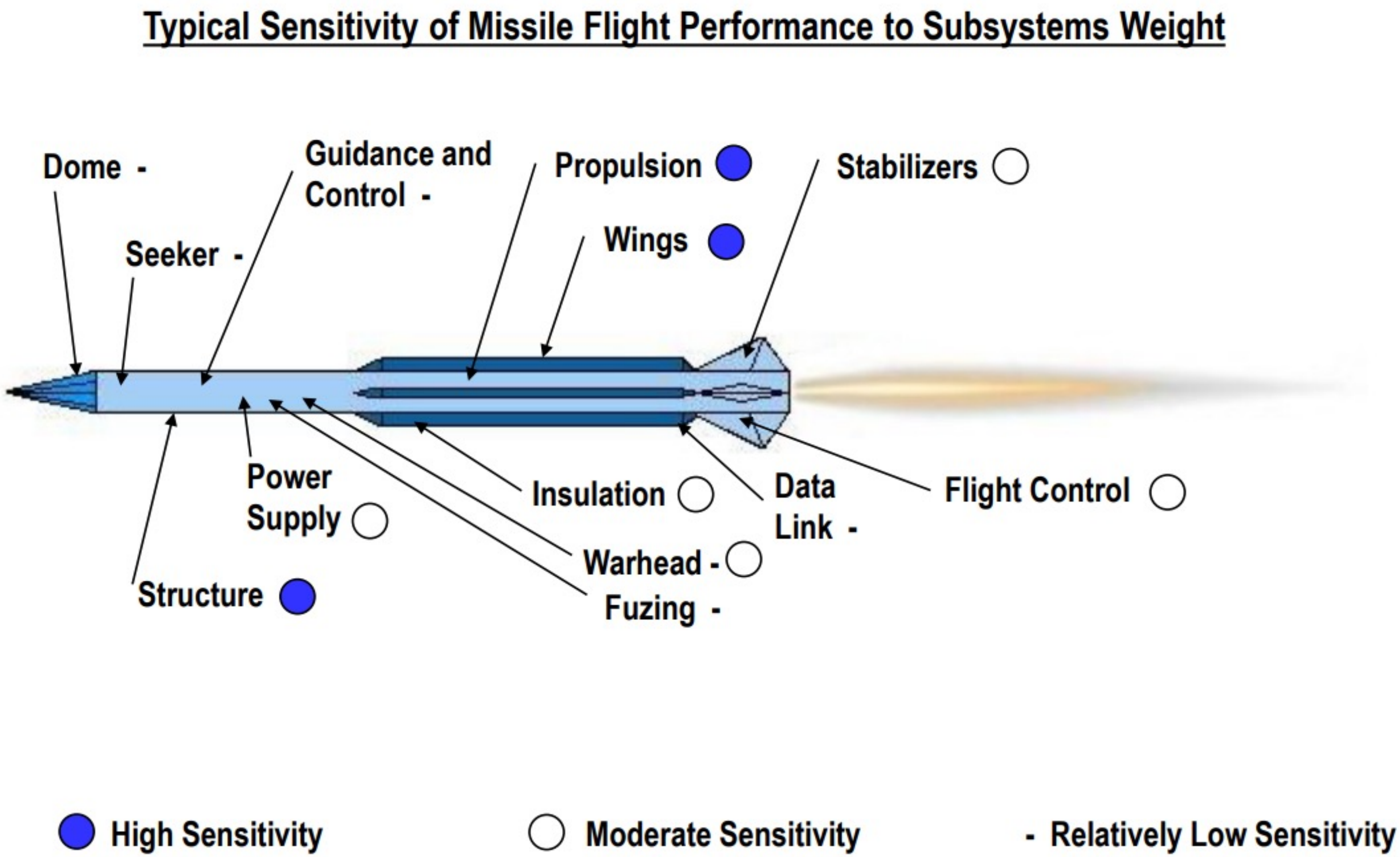
Missile Flight Performance / Trajectory is Driven by Forces (Aerodynamics, Propulsion, Weight)



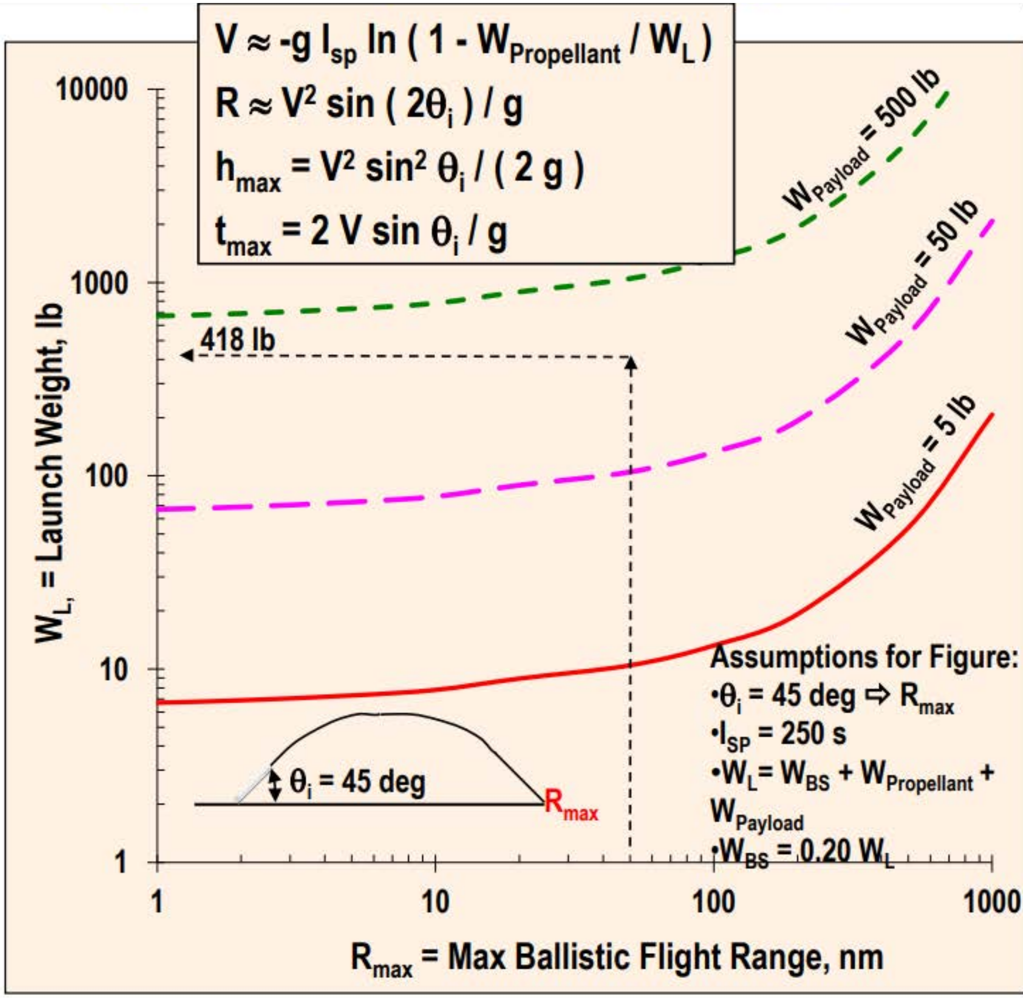
A Lightweight Missile Has Payoff



Weights of Missile Subsystems Often Drive Missile Flight Performance



Ballistic Missile Weight Driven by Range, Payload Weight, Propellant Weight, and Specific Impulse



Assumptions:

- $T \gg W \sin\gamma$ and $T \gg D$ during thrust and $W \sin\gamma \gg D$ ⇨ Optimistic Result
- Flat, Non-rotating Earth

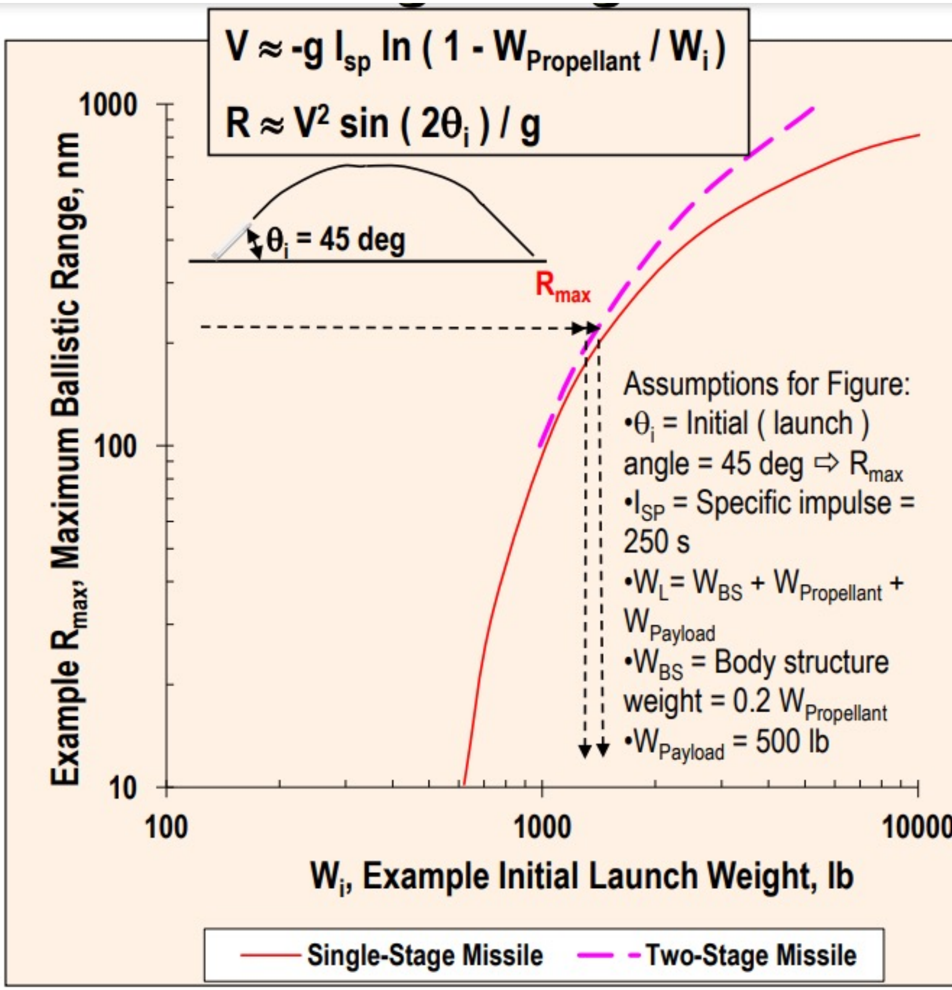
Nomenclature:

- V = Burnout Velocity
- g = gravitational Constant = 32.2 ft / s^2
- I_{SP} = Specific Impulse
- $W_{Propellant}$ = Propellant Weight
- W_L = Launch Weight
- R = Ballistic Flight Range
- θ_i = Initial (Launch) Angle
- h_{max} = Max Altitude
- t_{max} = Max Time of Flight
- W_{BS} = Weight of Body Structure (Motor Case + Airframe)
- $W_{Payload}$ = Payload Weight (Warhead, Guidance)

Example: R = 50 nm = 303800 ft, θ_i = 45 deg, $W_{Payload}$ = 200 lb, I_{SP} = 250 s, $W_{BS} = 0.2\, W_L$

- $V = [\, 303800\, (\, 32.2\,) / (\, 1\,)]^{1/2} = 3128$ fps
- $W_L = W_{BS} + W_{Propellant} + W_{Payload} = 0.20\, W_L + [\, 1 - e^{-3128 / [\, 32.2\, (\, 250\,)]}\,]\, W_L + 200 = 0.20\, W_L + 0.322\, W_L + 200 \Rightarrow W_L = 418$ lb (190 kg)

Staging Provides Range / Weight Payoff for Long Range Ballistic Missiles



Assumptions:

- $T \gg W \sin\gamma$ and $T \gg D$ during thrust and $W \sin\gamma \gg D$ ⇨ Optimistic Result
- Flat, Non-rotating Earth
- For Multi-Stage Missile of $(W_i)_{Min}$: $\Delta V_1 = \Delta V_n$, where ΔV_n = Incremental Velocity of Stage n
- Negligible Weight for Interstage

Example: Two-Stage Missile with Minimum Weight and R_{max} = 200 nm = 1.216 x 10^6 ft

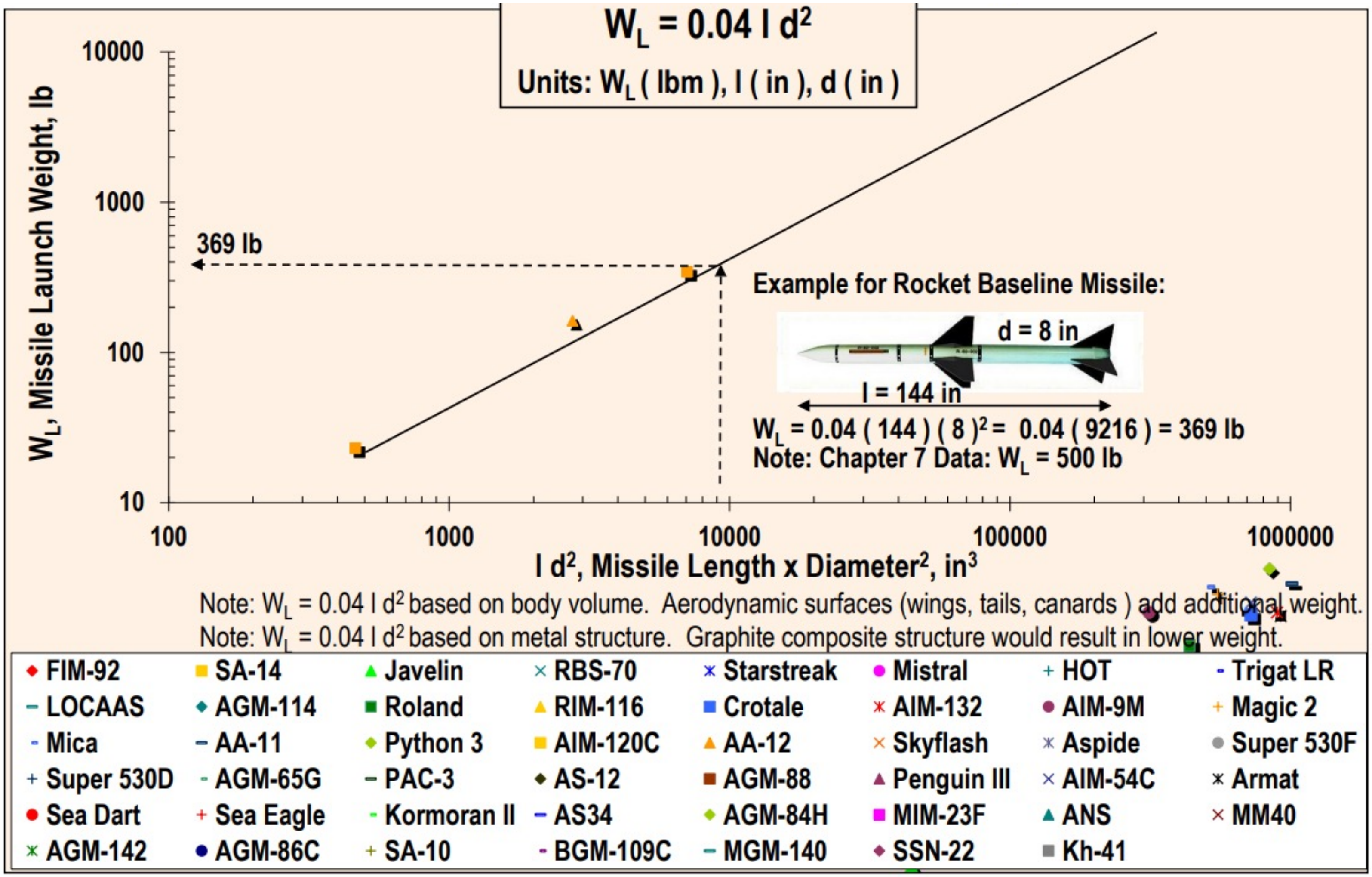
- **Assume θ_i = 45 deg, I_{SP} = 250 sec, $W_{Payload}$ = 500 lb, W_{BS} = 0.2 $W_{Propellant}$**
- **$V = [(32.2) (1.216 \times 10^6)]^{1/2} = 6251$ ft / s**
- **$\Delta V_1 = \Delta V_2 = V / 2 = 3125$ ft / s**
- **$W_{i,2ndStage} = W_{Payload} + W_{BS} + W_{Propellant} = 500 + 52 + 262 = 814$ lb**
- **$W_{i,1stStage} = W_{BS} + W_{Propellant} = 85 + 427 = 512$ lb**
- **$W_i = W_{i,1stStage} + W_{i,2ndStage} = 814 + 512 = 1326$ lb**

Example: One Stage Missile, R = 200 nm

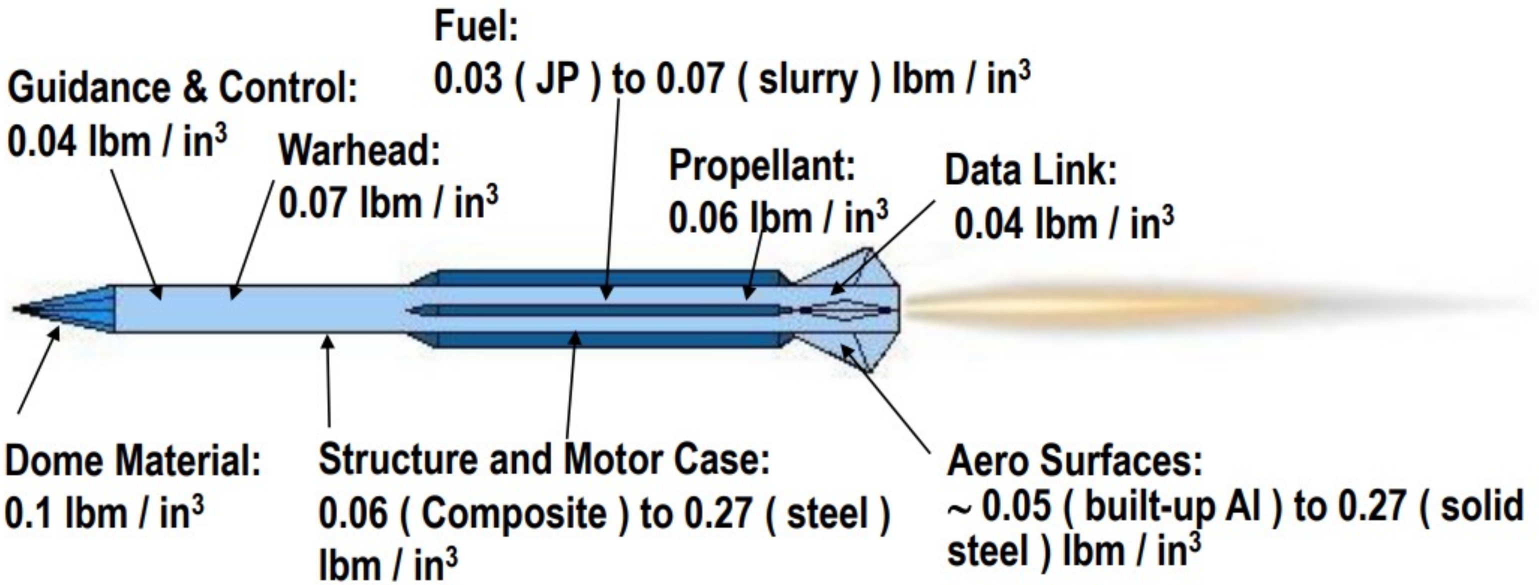
- **$V = 6251 = - 32.2 (250) \ln [1 - W_{Propellant} / (W_{Propellant} + 0.2\, W_{Propellant} + 500)] \Rightarrow W_{Propellant} = 767$ lb $\Rightarrow W_i = 1420$ lb**

Note: 200 nm Two-Stage Missile Is 7% Lighter than One-Stage Missile (1326 lb vs 1420 lb)

A First-Order Estimate of Missile Weight can be Derives from Bodt Geometry Dimensions

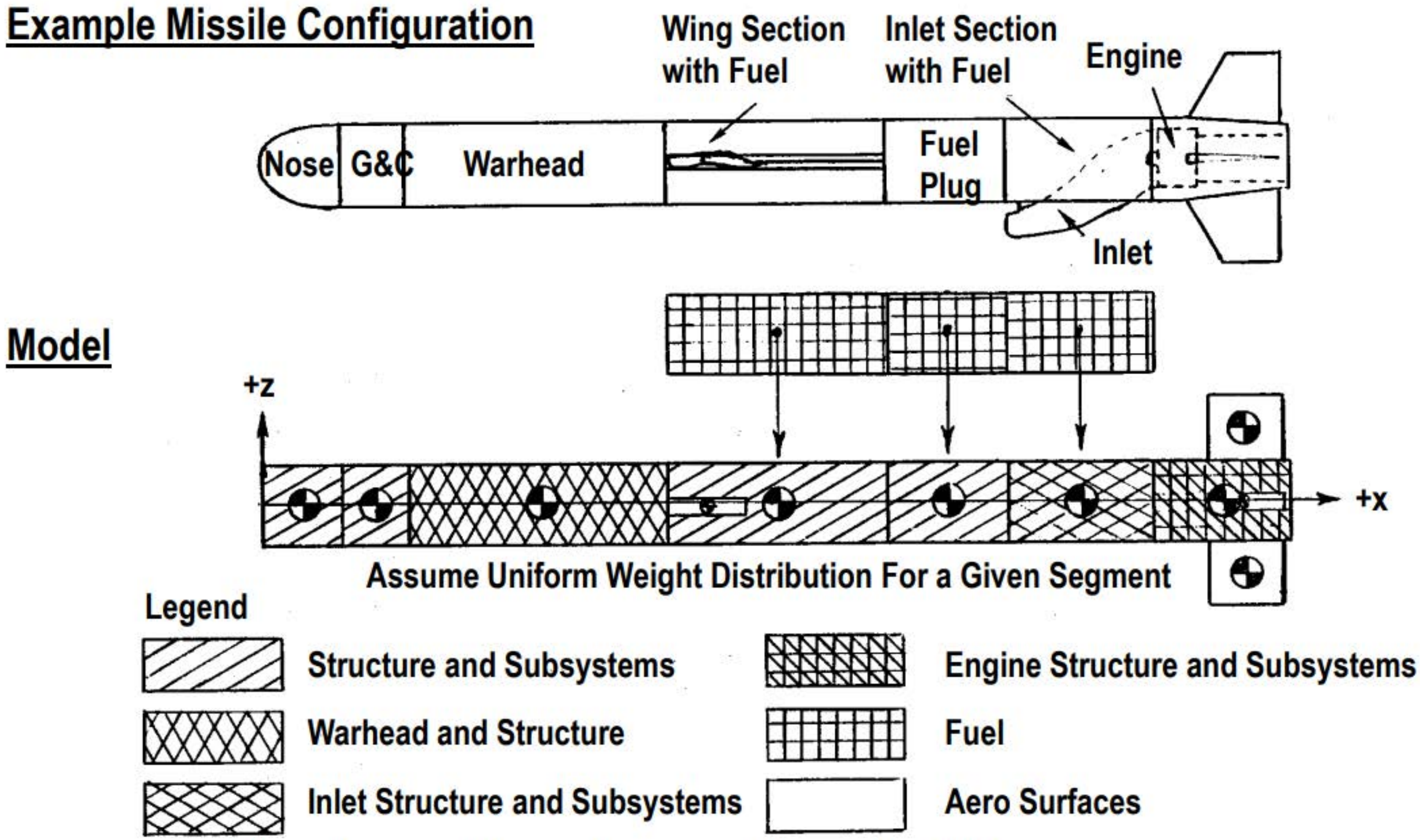


Most Subsystems for Missiles Have a Weight Density of about 0.05 lmb/in^3



Note:
Subsystem Weight = Subsystem Density x Subsystems Volume
Missile Weight = Σ Subsystems Weights
Missile Density ~ 1.4 x Density of Water (0.05 versus 0.0361 lbm / in^3, 1384 versus 997 kg / m^3)

Modeling Missile Weight, Balance, and Moment-of-Inertia is Based on a Build-up of Subsystems

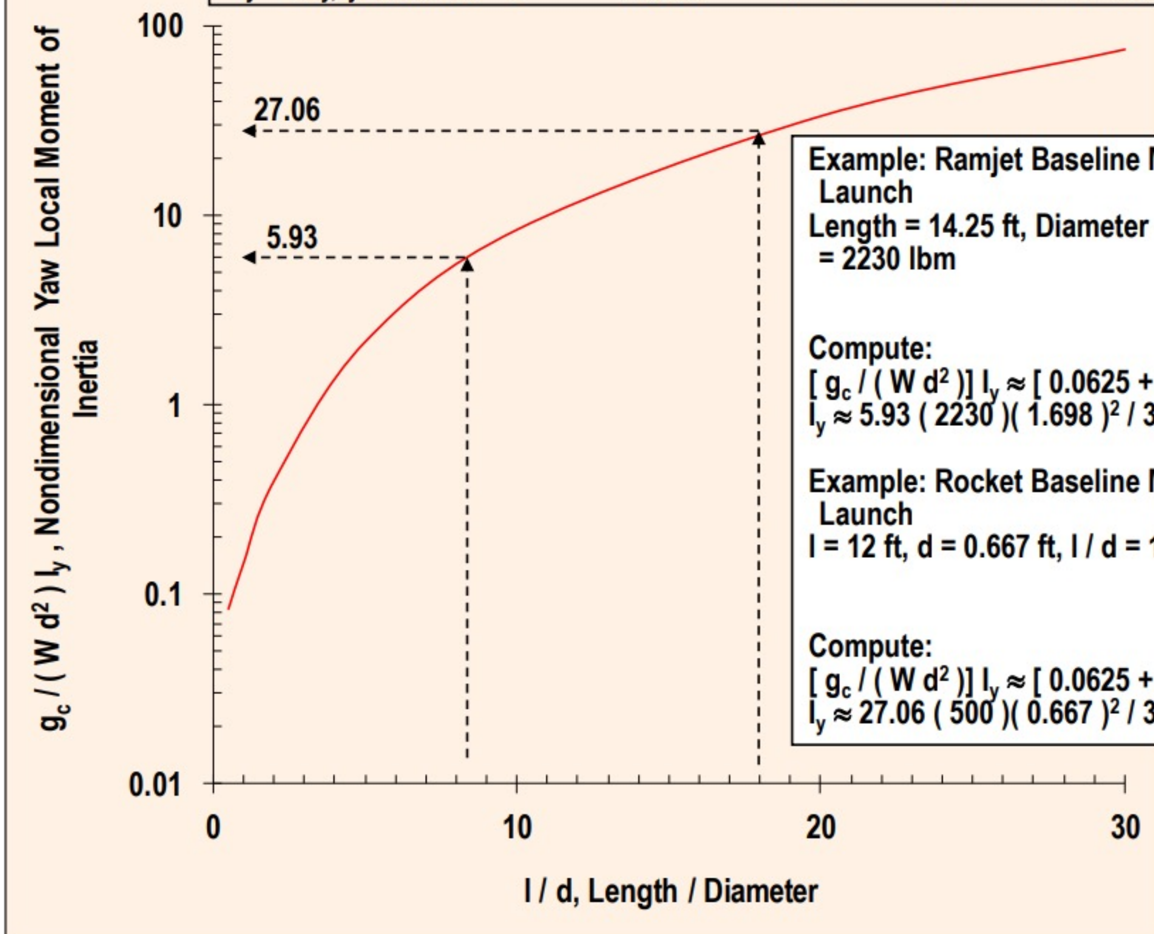


$x_{CG} = \Sigma \; (x_{subsystem1} W_{subsystem1} + x_{subsystem2} W_{subsystem2} + \ldots) / W_{total}$

$I_Y = \Sigma \; [(I_{y,subsystem1})_{local} + W_{subsystem1} (x_{subsystem1} - x_{CG})^2 / g_c + (I_{y,subsystem2})_{local} + W_{subsystem2} (x_{subsystem2} - x_{CG})^2 / g_c + \ldots]$

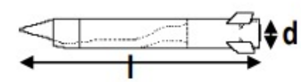
Missile Moment-of-Inertia is Driven by Length Diameter, and Weight

$I_y \approx (I_{y,cylinder})_{local} = [W d^2 / g_c] [(1 / 16) + (1 / 12) (l / d)^2]], g_c = 32.2$ lbm / slug



Example: Ramjet Baseline Missile Moment-of-Inertia at Launch

Length = 14.25 ft, Diameter = 1.698 ft, ⇒ l / d = 8.39, Weight = 2230 lbm



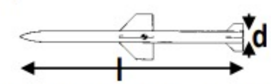
Compute:

$[g_c / (W d^2)] I_y \approx [0.0625 + 0.0833 (8.39)^2] = 5.93 \Rightarrow$

$I_y \approx 5.93 (2230)(1.698)^2 / 32.2 = 1183$ slug-ft² (1603 kg-m²)

Example: Rocket Baseline Missile Moment-of-Inertia at Launch

l = 12 ft, d = 0.667 ft, l / d = 18, W = 500 lbm



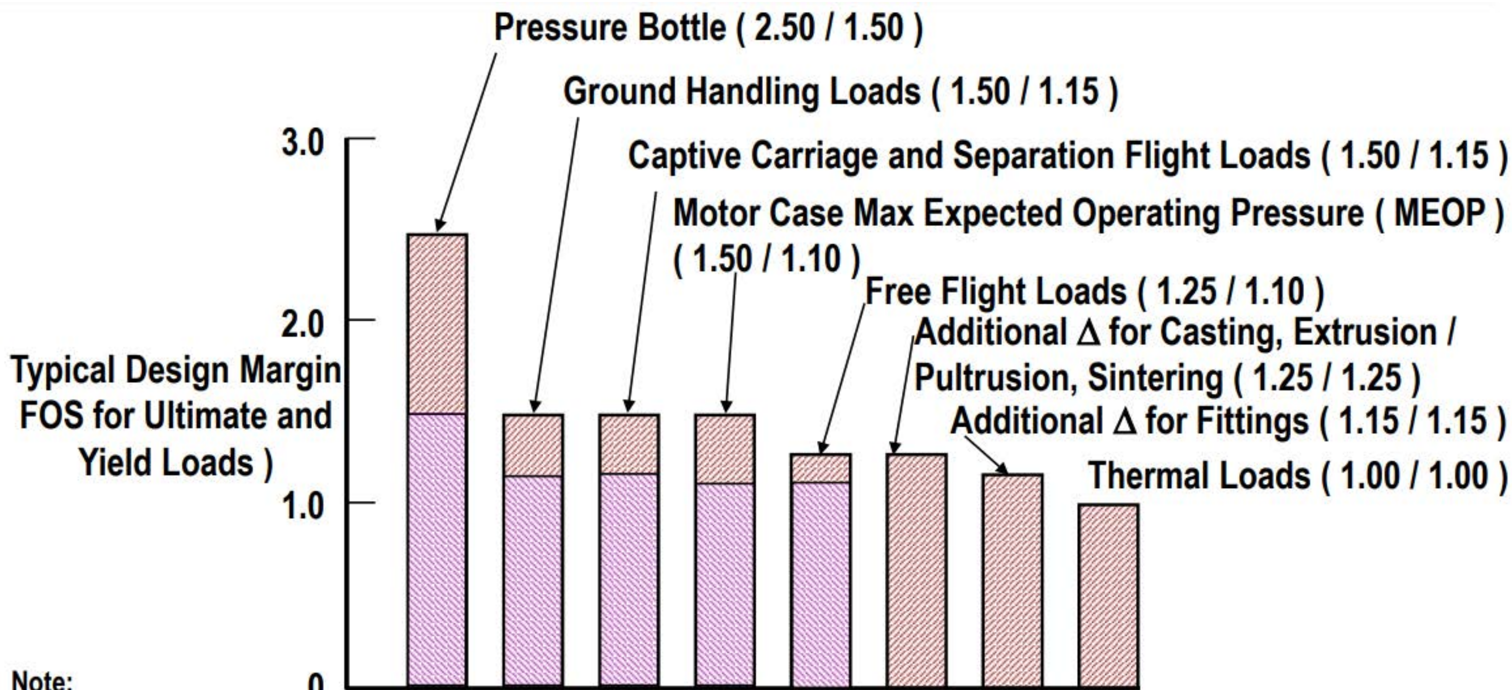
Compute:

$[g_c / (W d^2)] I_y \approx [0.0625 + 0.0833 (18)^2] = 27.06 \Rightarrow$

$I_y \approx 27.06 (500)(0.667)^2 / 32.2 = 187$ slug-ft² (253 kg-m²)

Note: For a cruciform missile, yaw and pitch moments of inertia are nearly equal ($I_y \approx I_x$)

Structure Design Factor of Safety Must be Greater for Hazardous Subsystems / Flight Conditions

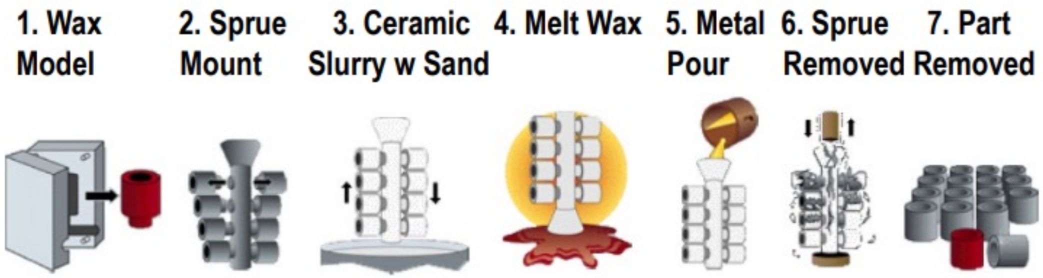


Note:

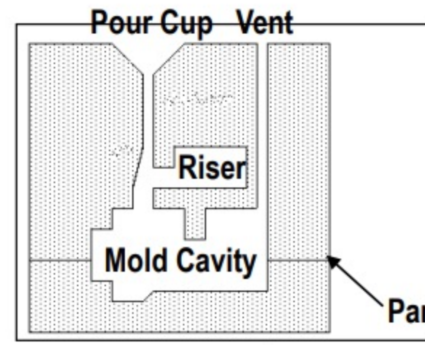
- $(FOS)_{Ultimate}$ = design ultimate (failure) load / predicted ultimate load, $(FOS)_{Yield}$ = design yield load / predicted yield load.
- MIL STDs include environmental (HDBK-310, NATO STANAG 4370, 810G, 1670A), strength and rigidity (8856), pressurization (1522A), and captive carriage (8591).
- The entire environment (e.g., manufacturing, transportation, storage, ground handling, captive carriage, launch separation, post-launch maneuvering, terminal maneuvering) must be examined for driving conditions in structure design.
- Δ FOS for casting, extrusion / pultrusion, and sintering is expected to be reduced / eliminated in future as technologies mature.
- Reduction in required factor of safety is expected as analysis accuracy improves will result in reduced missile weight / cost.

Examples of Missile Structures Manufacturing Processes

Casting

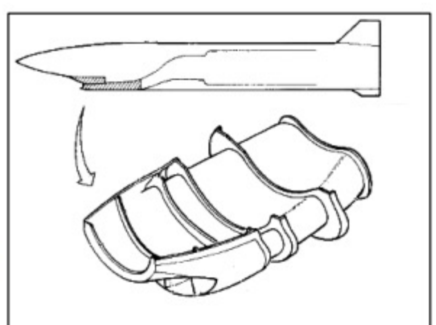


Investment Casting Process



Permanent Mold Casting Process

Tactical Tomahawk Aluminum Body Casting



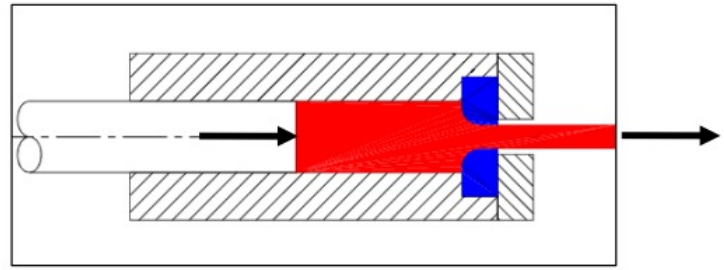
ASALM Titanium Inlet Casting

Video of Investment Casting

Examples of Missile Structue Manufacturing Processes (cont)

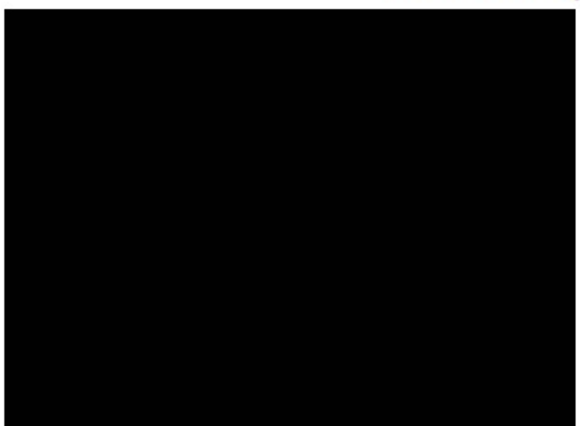
Forming

Video of Forging, Ring / Strip Rolling

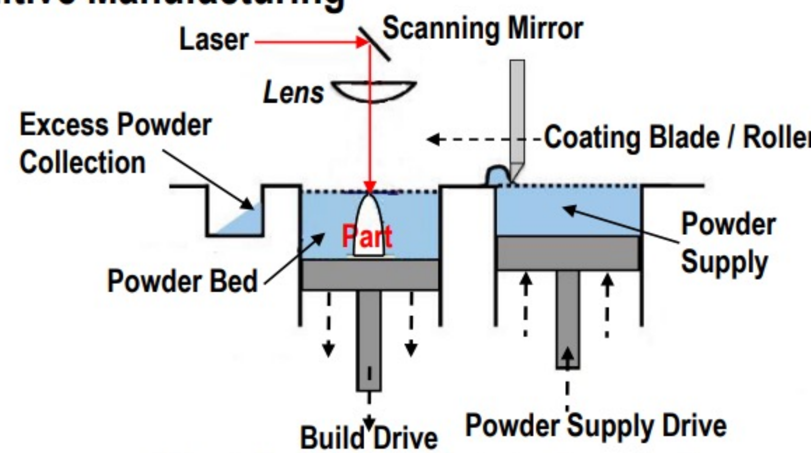


Extrusion Process

3D Printing / Additive Manufacturing



Video of 3D Printing Using DMLS (Courtesy of Solid Concepts)



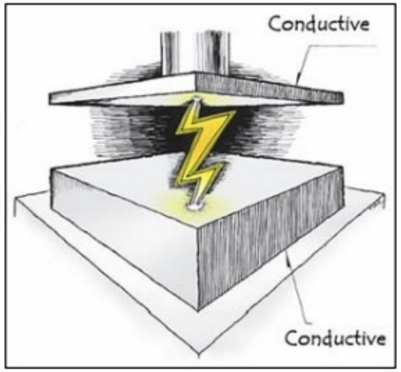
Direct Metal Laser Sintering Powder Process

Examples of Missile Structue Manufacturing Processes (cont)

Machining

Video of Cutting, Milling, Drilling, EDM

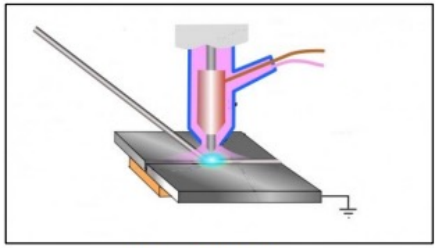
Laser Cutting



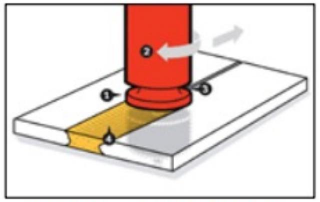
Electrical Discharge Machining

Welding

Video of Resistance / Arc, Laser, Friction Welding



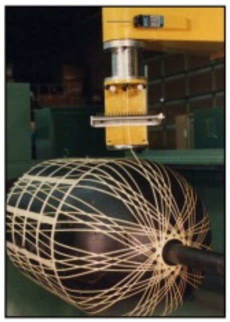
Arc Welding

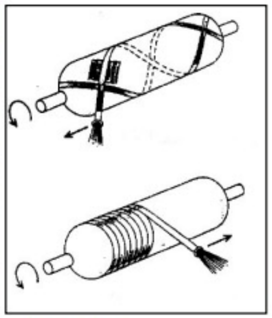


Friction Stir Welding

Examples of Missile Structue Manufacturing Processes (cont)

Composite Filament Winding

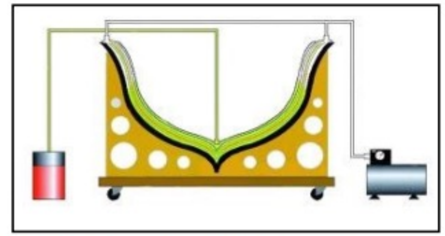




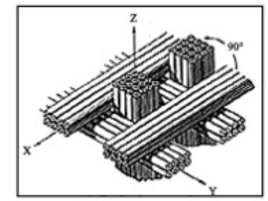
Filament Winding

PAC-3 Quartz Composite Radome

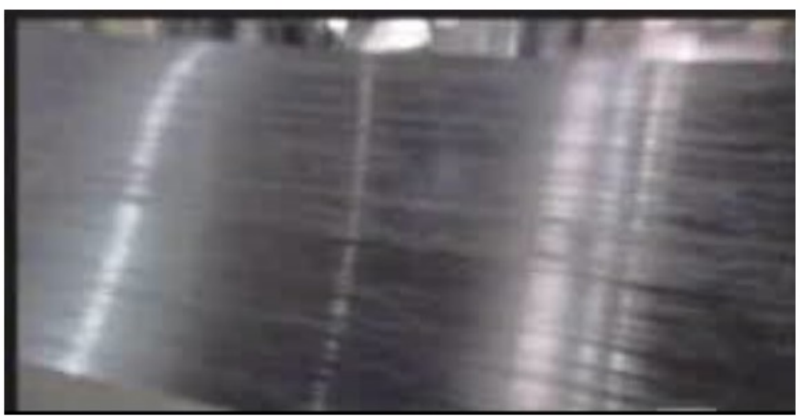
Resin Transfer Molding



Resin Transfer Molding Process



3D Fiber Orientation

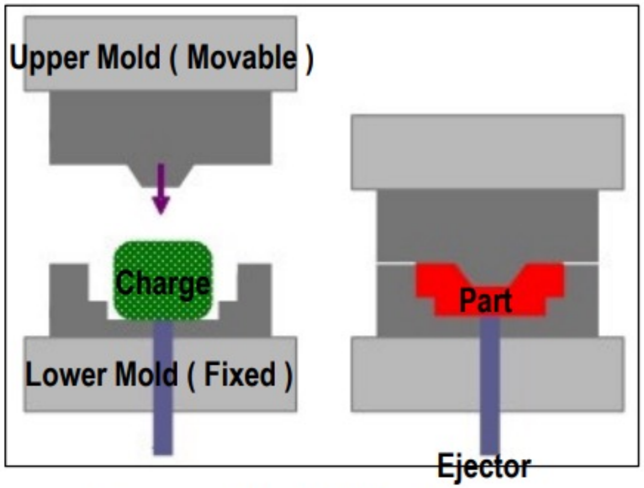


Video of Carbon Fiber Manufacturing

Video of Composite Filament Winding, RTM, Vacuum Bagging

Examples of Missile Structue Manufacturing Processes (cont)

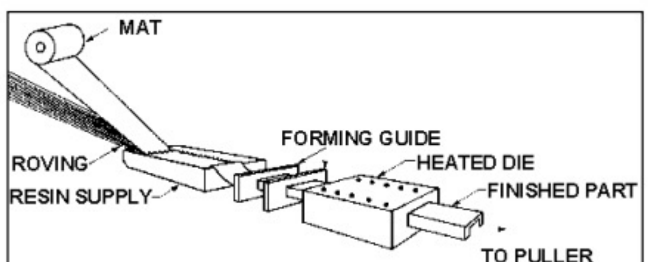
Compression Molding



<u>Compression Molding Process</u>

Video of Compression Molding
(Courtesy of Carbone Forge)

Pultrusion



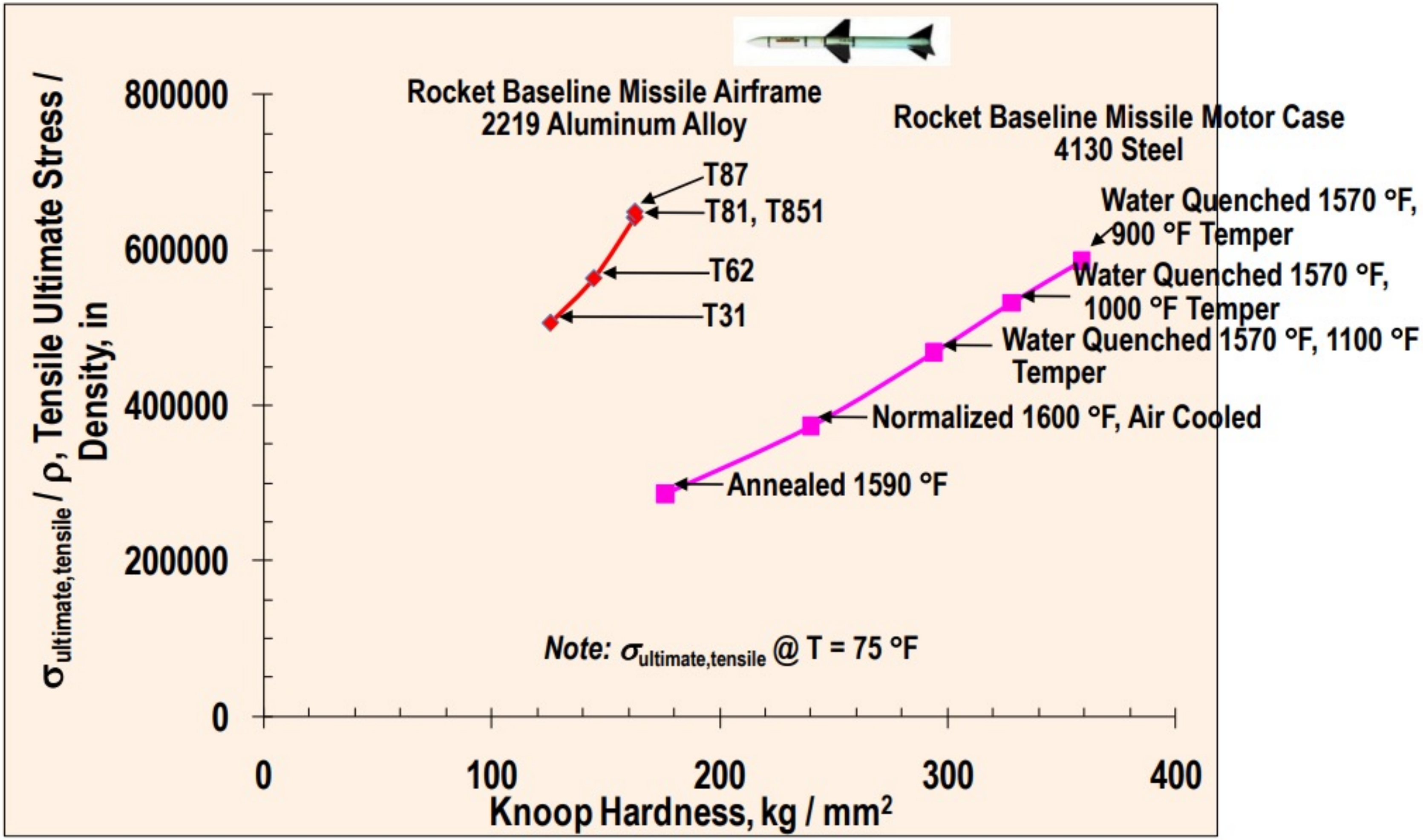
<u>Pultrusion Process</u>

Mechanical Fastner Versus Adhesive Bonding is Tradeoff for Structural Joint Attachment

Type of Joint Attachment	Max Load	Thermal Stress Metal – Metal	Thermal Stress Graphite – Graphite	Thermal Stress Metal - Graphite	Fatigue	Inspection / Disassembly
Mechanical Fastener 	●	○	○	◡	○	●
Adhesive Bonding 	◡	◒	●	○	●	◡

● Superior ◒ Good ○ Average ◡ Poor

Increasing Metal Hardness Increases Strength, but Machining is more Difficult / Expensive



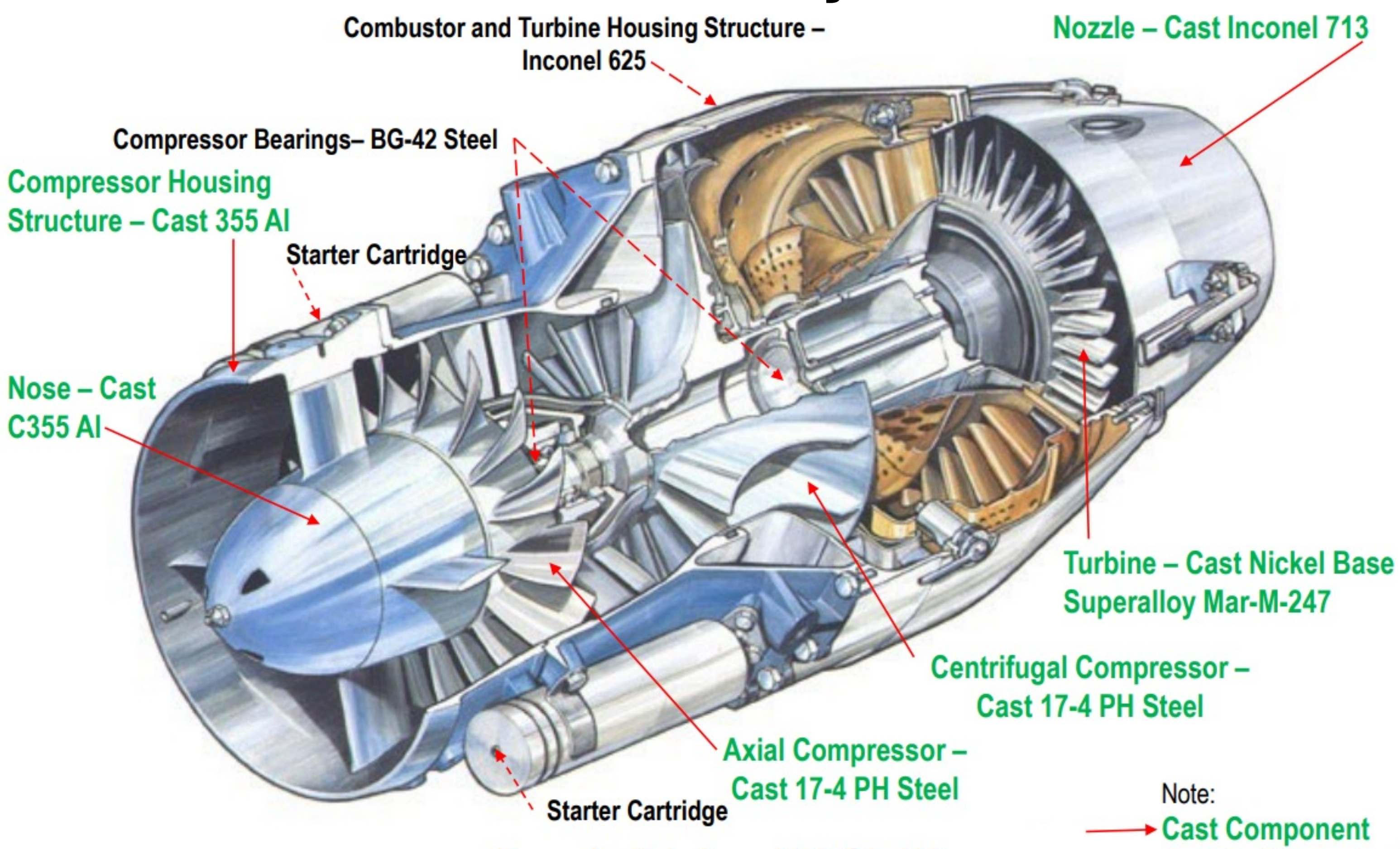
Missile Structure Parts Count is Driven by Manufacturing Process

Missile Airframe Geometry Alternatives	Missile Structure Concept Alternatives	Structure Manufacturing Process Alternatives										
		Graphite Composites						Metals				
		Vacuum Assist RTM	Compression Mold	Filament Wind	Pultrusion	Thermal Form	Vacuum Bag / Autoclave	Cast	3D Print / Additive	High Speed Machine	Forming	Strip Laminate
Non-Axisymmetric Body Airframe	Monocoque	●	◒	●	●	◒	◒	●	●		◒	
	Integrally Hoop Stiffened	●	◒				◒	●	●	–		
	Integrally Longitudinal Stiffened	●	◒		●		◒	●	●	–		
Axisymmetric Body Airframe	Monocoque	●		●	●		●	●	●		●	●
	Integrally Hoop Stiffened	●					◒	●	●	◒		
	Integrally Longitudinal Stiffened	●			●		◒	●	●	◒		
Aerodynamic Surface	Solid	●	●			●	●	●	●	●	●	
	Sandwich	◒	◒				◒		●		○	

Note: Manufacturing process cost is a function of recurring cost (unit material, unit labor) and non-recurring cost (tooling)

Note: ● **Very Low Parts Count** ◒ **Low Parts Count** ○ **Moderate Parts Count** – **High Parts Count**

Missile Turbojet engine Parts are Often Castings: Lower Parts Count → Lower Cost and Higher Reliability



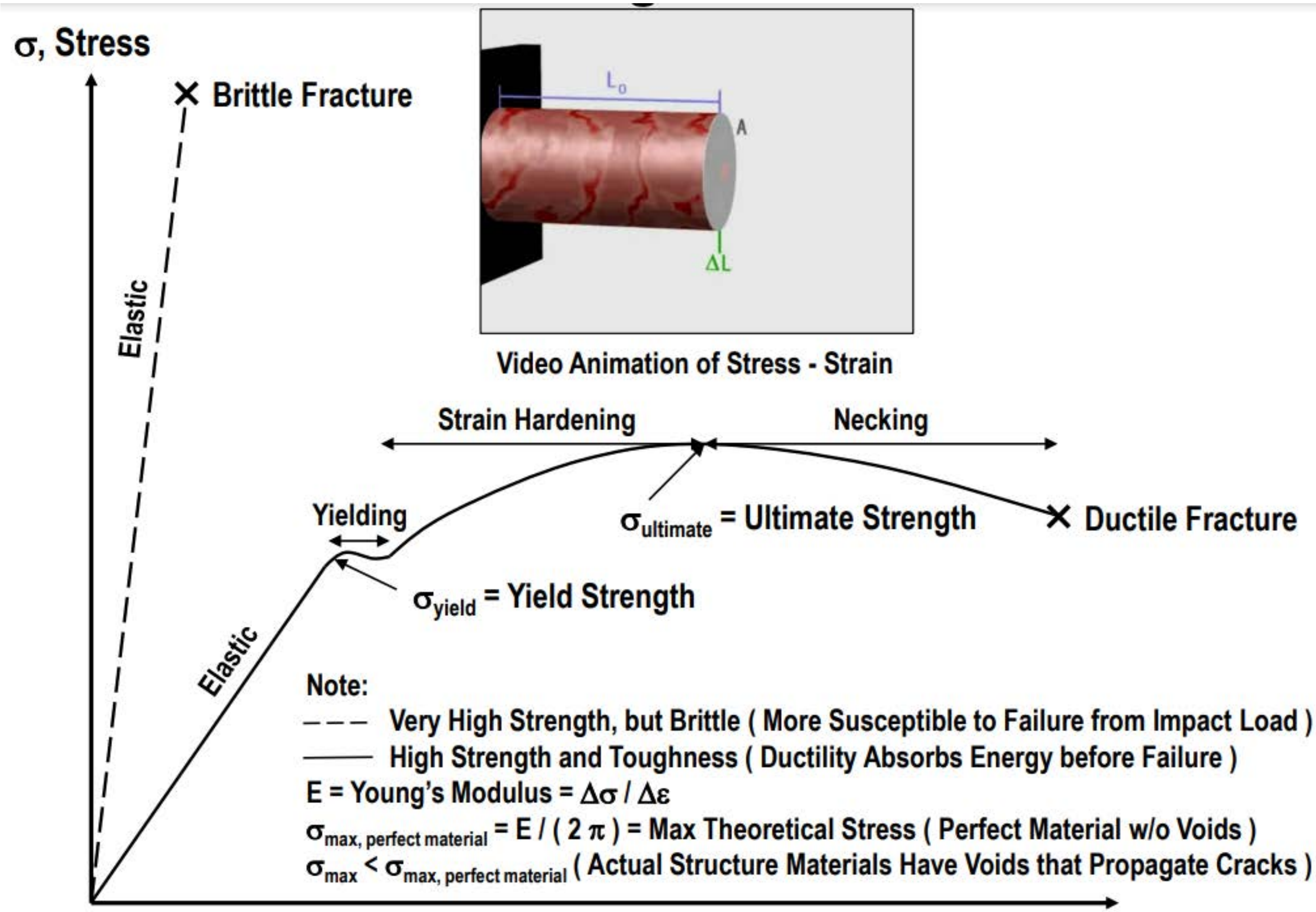
Example: Teledyne J402-CA-400

Missile Airframe Material Alternatives Include Aluminum, Steel, Titanium, and Composite

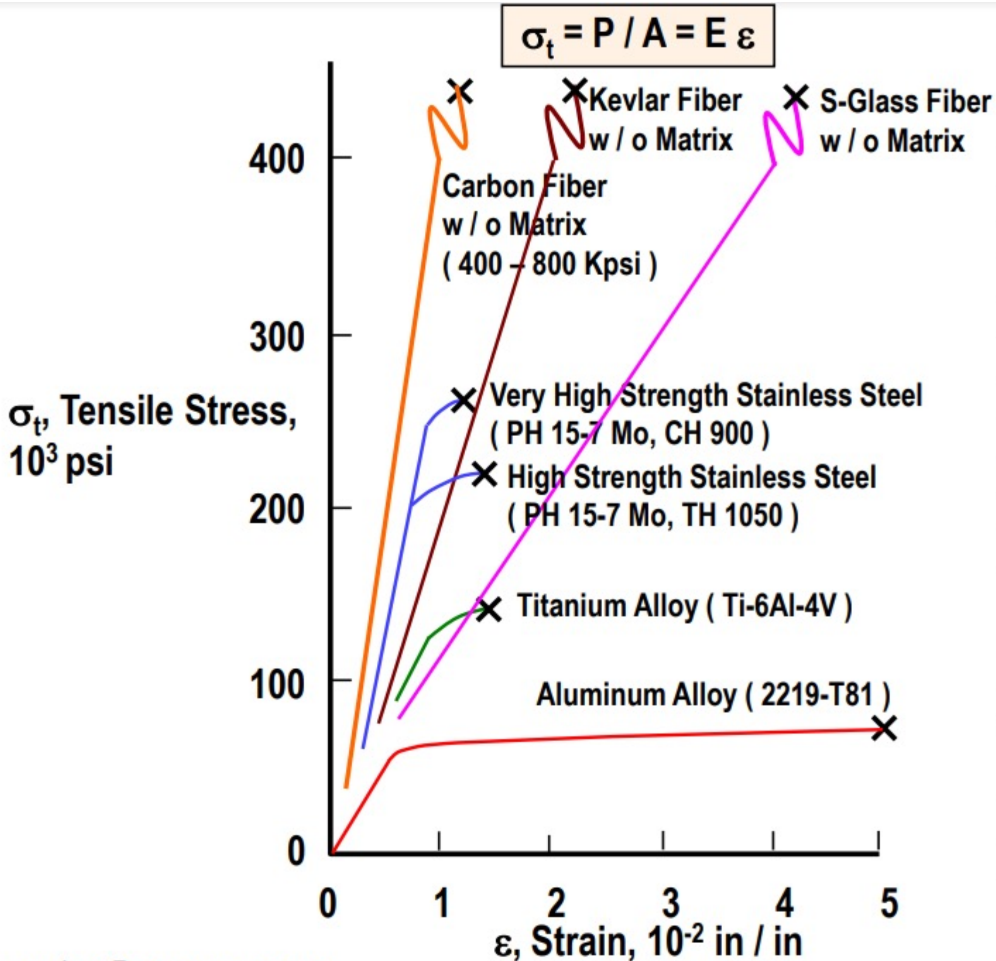
Type	Material	Tensile Stress (σ_{TU} / ρ)	Buckling Stability ($\sigma_{Buckling} / \rho$)	Max Short – Life Temp	Thermal Stress	Joining	Fatigue	Cost	Weight
Metallic ↓	Aluminum 2219	○	◒	– ○	–	◒	–	● ○	○
	Steel PH 15-7 Mo	◒	–	●	○	●	◒	● ○	–
	Titanium Ti-6Al-4V	◒	○	●	◒	○	◒	–	○
Composite ↓	S994 Glass / Epoxy and S994 Glass / Polyimide	◒	– ○	○	◒	○	●	●	◒
	Glass or Graphite Reinforce Molding	–	– ○	○	◒	○	●	○	◒
	Graphite / Epoxy and Graphite Polyimide	●	○	○ ◒	●	–	●	–	●

Note: ● Superior ◒ Good ○ Average – Poor

Missile Structure Drivers Include Strength and Toughness



Example of Strength – Elasticity Comparison of Missile Structure Material Alternatives



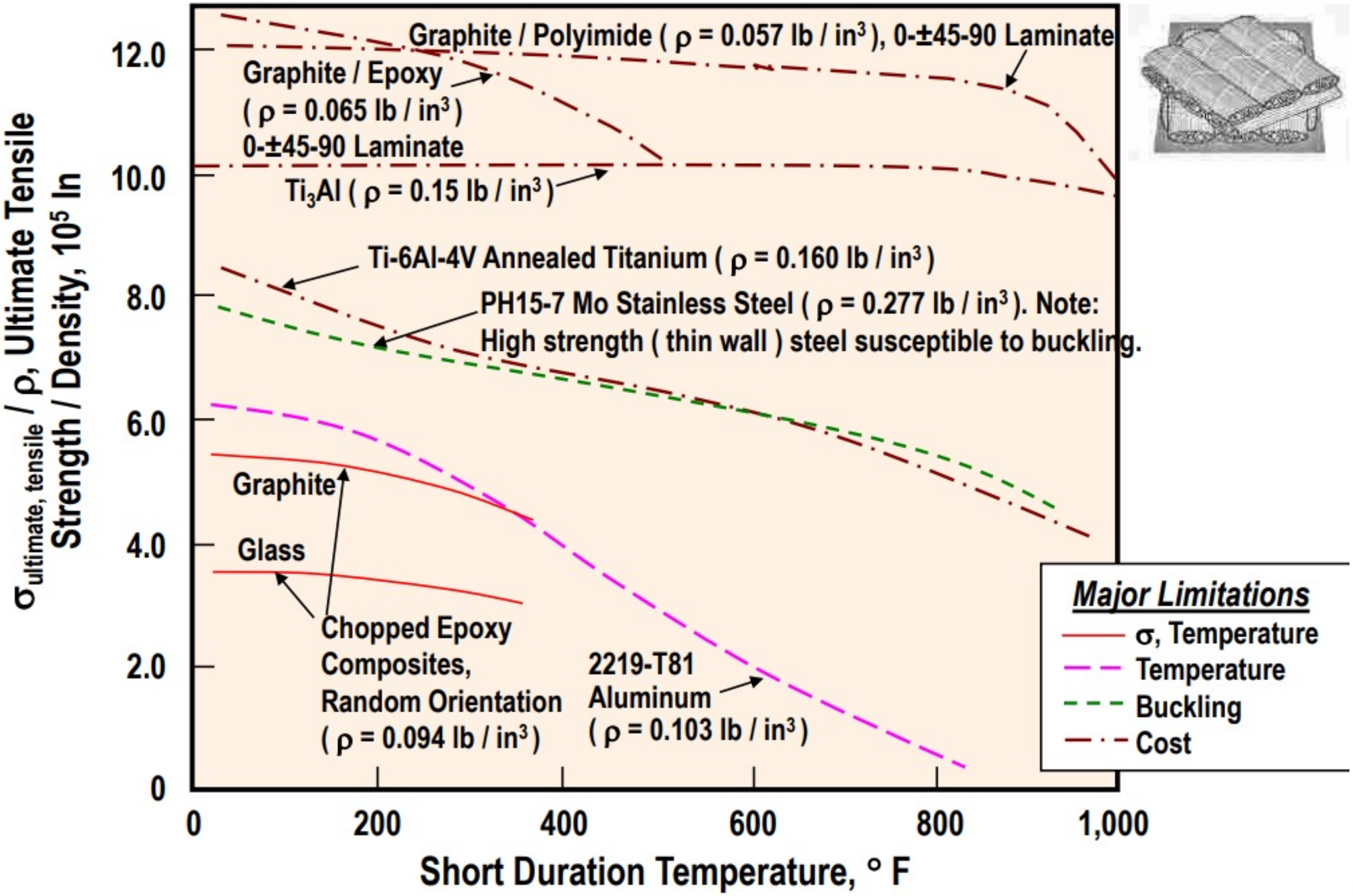
Note:

- Carbon Composites:
  - High strength
  - Low strain
  - Very small diameter fibers
  - Long length / continuous fibers
  - Unidirectional
  - High modulus of elasticity
  - Very elastic
  - No yield before failure
  - Non forgiving failure
  - Susceptible to moisture
  - Susceptible to lightning / ESD
  - Susceptible to impact load
  - Many resins are toxic
- Metals:
  - More ductile, yield before failure
  - Allow adjacent structure to absorb load
  - Resist crack formation
  - Resist impact loads
  - More forgiving failure

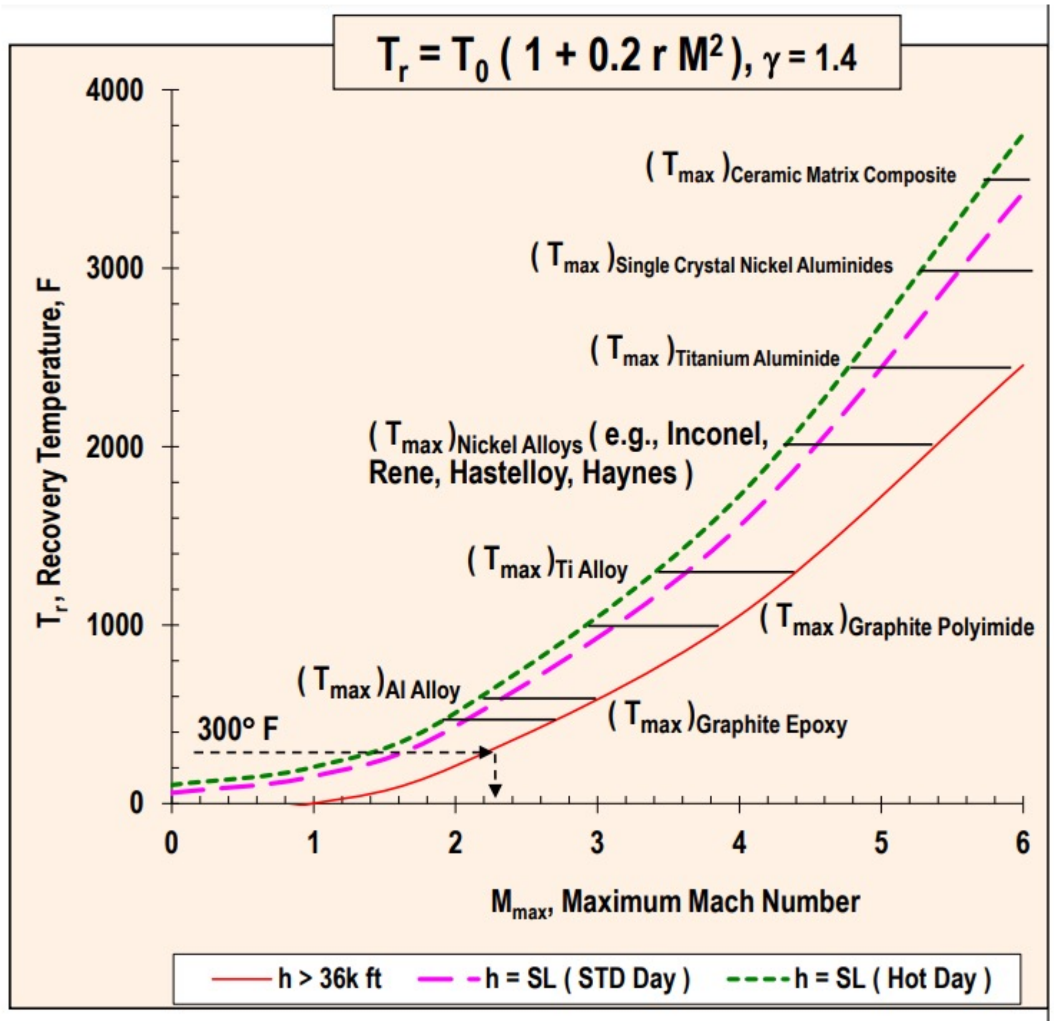
Assumption: Room temperature

Nomenclature: E = Young's modulus of elasticity, psi; P = Load, lb; ε = Strain, in / in; A = Area, in^2

Laminate Graphite Composite Provides a High Strength-to-Weight Structure



A High Speed Missile without External Insulation Requires High Temperature Structure



Note for Equation:

- T_r = Recovery Temperature, R
- T_0 = Free Stream Temperature, R
- r = Recovery Factor
- Laminar Boundary Layer ⇨ $r \approx 0.85$
- Turbulent Boundary Layer ⇨ $r \approx 0.9$
- Stagnation ⇨ $r \approx 1$
- M = Free Stream Mach Number

Assumptions for Figure:

- No External Insulation
- Thermal Soak ($T_r \approx T_{structure}$)
- T_{max} = Max Temperature of Material
- h > 36k ft ⇒ T_0 = 390 R = - 70° F
- h = SL (STD day) ⇒ T_0 = 519 R = 59° F
- h = SL (Hot Day) ⇒ T_0 = 563 R = 103° F
- Turbulent Boundary Layer r = 0.9

Example for Hot (Thermal Soak) Al Alloy Airframe with T_{max} = 300° F = 760 R, h > 36k ft (T_0 = 519 R), Turbulent Boundary Layer (r = 0.9)

- **$M_{max} = \{ 5 [(T_r / T_0) - 1] / r \}^{1/2} = \{ 5 [(760 / 390) - 1] / 0.9 \}^{1/2} = 2.3$**

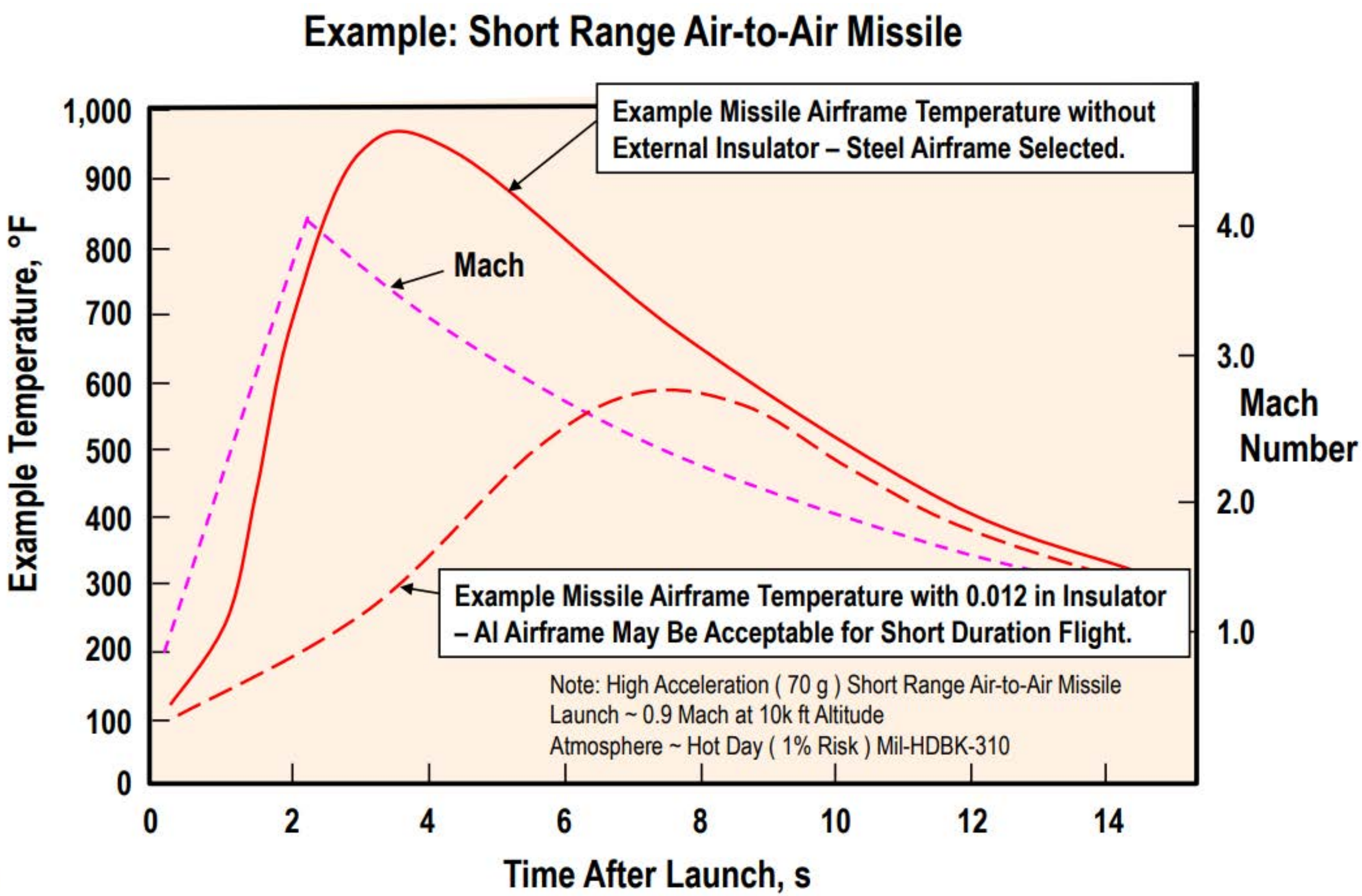
Examples of Missile Structure – Insulation Concepts

Example Structure / Insulation Concepts	Concept	T_{max}	k	c	ρ	α
Hot Metal Structure (e.g., Al Heat Sink) without Insulation	1	300 - 600	0.027	0.22	0.103	0.000722
Hot Metal Structure (e.g., Al Heat Sink)	2	300 - 600	0.027	0.22	0.103	0.000722
Internal Insulation (e.g., Min-K)		2000	0.0000051	0.24	0.012	0.00000106
Self-insulating Composite Structure (e.g., Graphite Polyimide)	3	1100	0.000109	0.27	0.057	0.00000410
Ext Insulation (e.g., Micro-Balloon Quartz)		1200	0.0000131	0.28	0.012	0.00000226
Cold Metal Structure (e.g., Al Heat Sink)	4	300 – 600	0.027	0.22	0.103	0.000722
Internal Insulation (e.g., Min-K)		2000	0.0000051	0.24	0.012	0.00000106

Note:

- Missiles use passive thermal protection (no active cooling) based on structure heat sink, insulation, and possibly phase change material (e.g., paraffin to cool electronics).
- Small thickness for insulation allows more propellant / fuel for diameter constrained missiles (e.g., VLS launcher).
- Weight and cost are application specific.
- T_{max} = max temp capability, °F; k = thermal conductivity, BTU / s / ft / °F; c = specific heat or thermal capacity, BTU / lbm / °F; ρ = density, lbm / in^3; α = thermal diffusivity = k / (ρ c), ft^2 / s
- Insulation thickness driven by thermal diffusivity = k / (ρ c). Insulation weight driven by k ρ / c.

External Structure Insulation has High Payoff for Short Duration Flight at High Mach Number

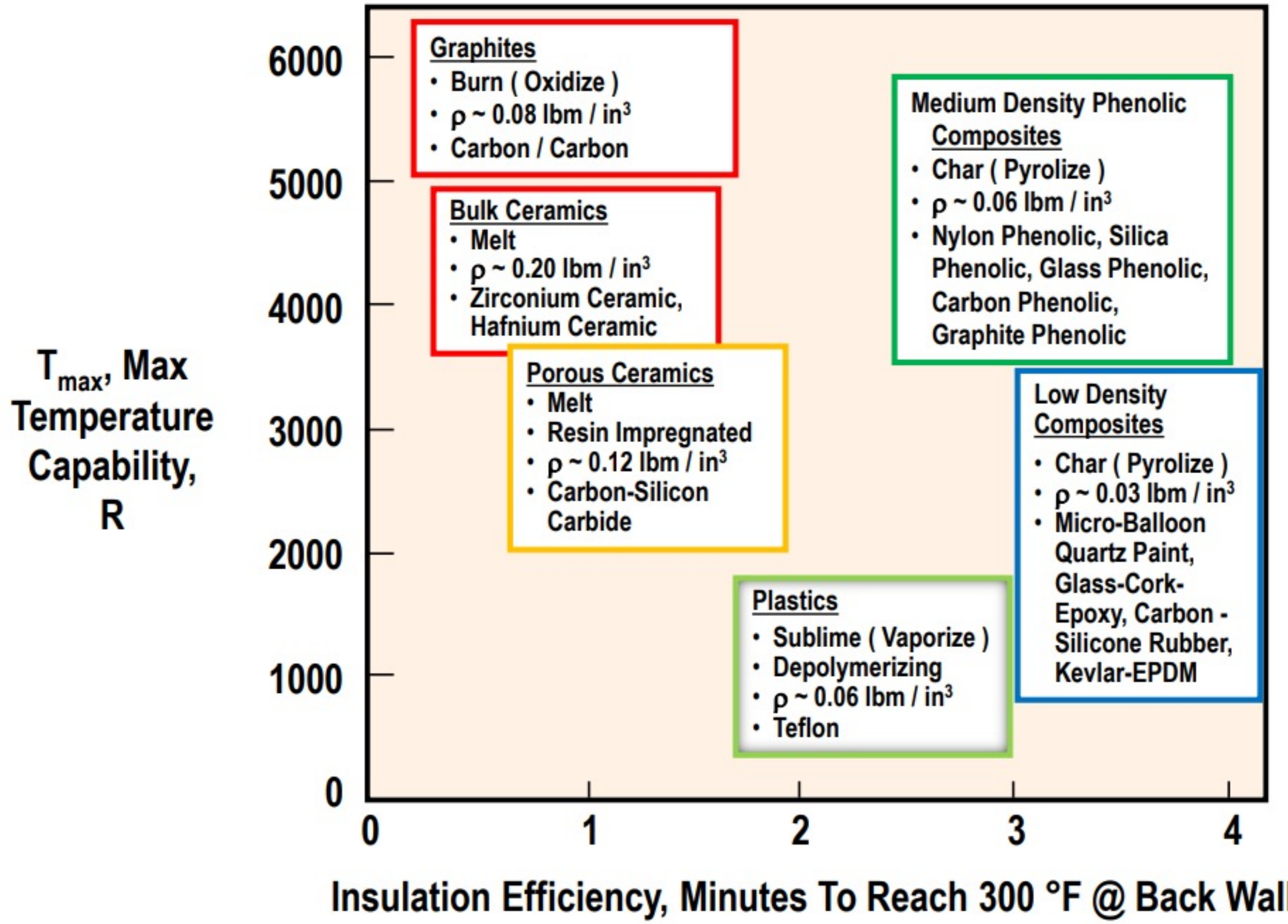


There are Many Considerations for High Temperature Missile Insulation

Type	Min Thickess	Max Temp	Max Mach	Min Weight	Strain / Shock	Strength	Out-Gassing	Cost
Phenolic Composites	◒	◒	●	○	○	○ ●	◒	○
Low Density Composites	●	○	○ ◒	◒	●	○	○	○ ●
Plastics	○	-	-	○	●	-	-	●
Porous Ceramics	-	○	◒	-	-	●	●	○
Bulk Ceramics	-	◒	●	-	-	●	●	○ ●
Graphites	-	●	●	○	-	●	●	-

Note: ● Superior ◒ Good ○ Average - Poor

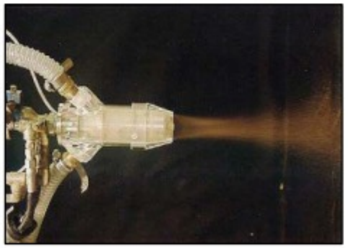
Max Temperature and Insulation Efficiency are Drivers for High Temperature Insulation



Note: Assumed Weight Per Unit Area of Insulator / Ablator = 1 lb / ft^2

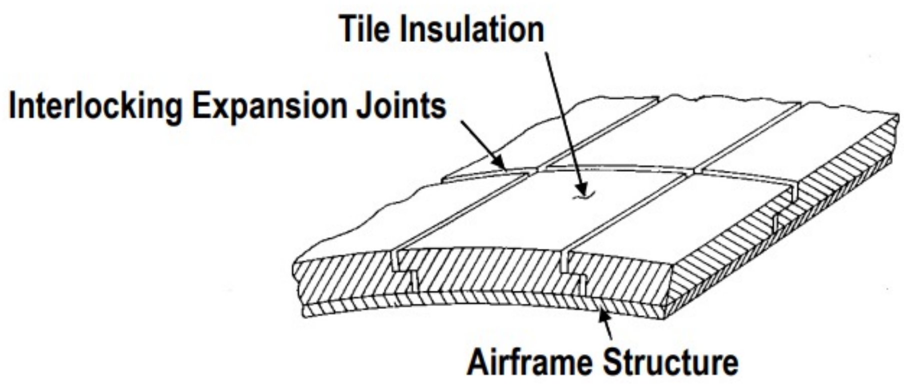
Required Insulation Thickness is a Consideration for Integrating External Insulation with Airframe

- Paint Insulation Usually Best if Insulation Thickness < ≈ 0.5 in
  - Spray or Trowel Multiple Coats

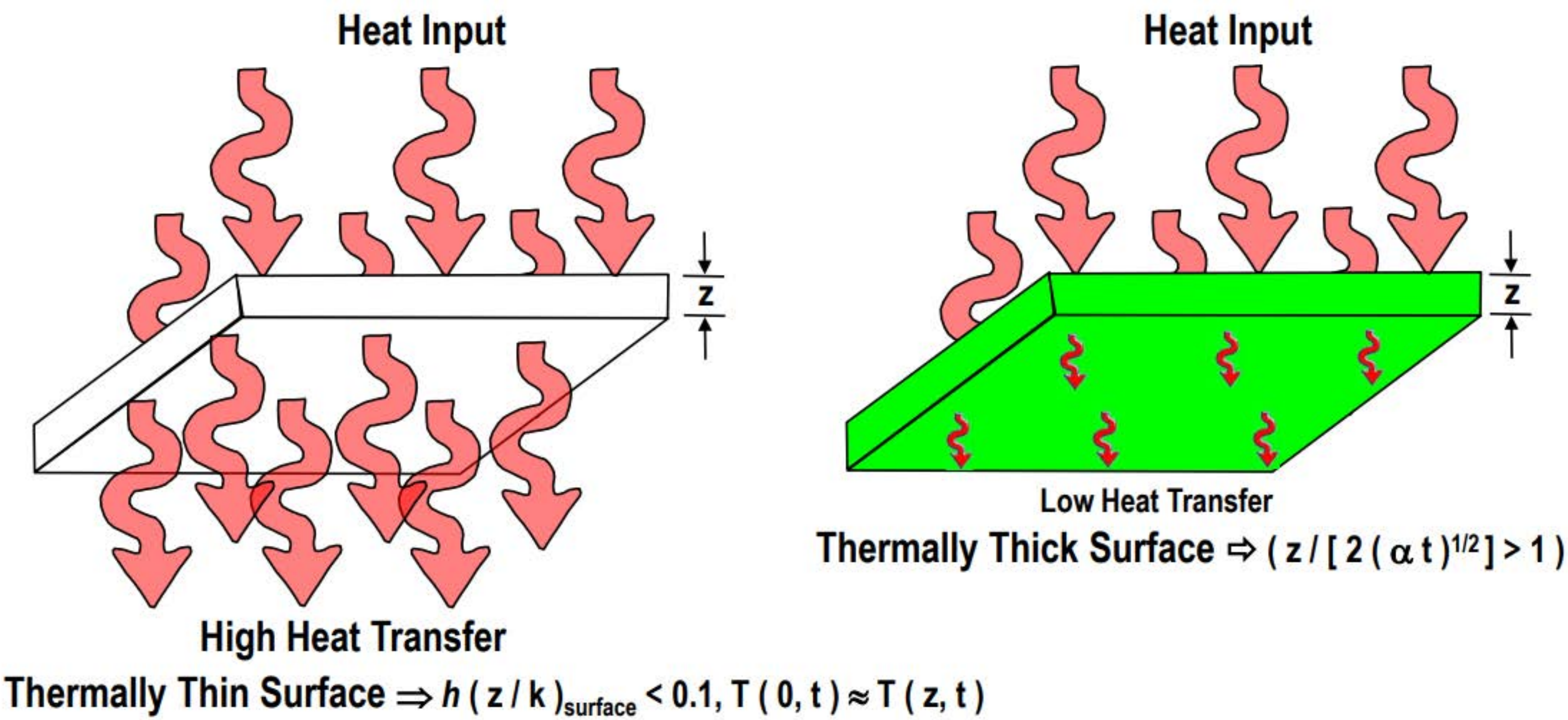


Spray-on Insulation for NASA SLS

- Tile Insulation Usually Best if Insulation Thickness > ≈ 0.5 in
  - Bond Insulation to Airframe Using High Temperature Adhesive (e.g., Silicone)
  - Insulation Expansion Joints May Be Required to Alleviate Thermal Stress from Airframe

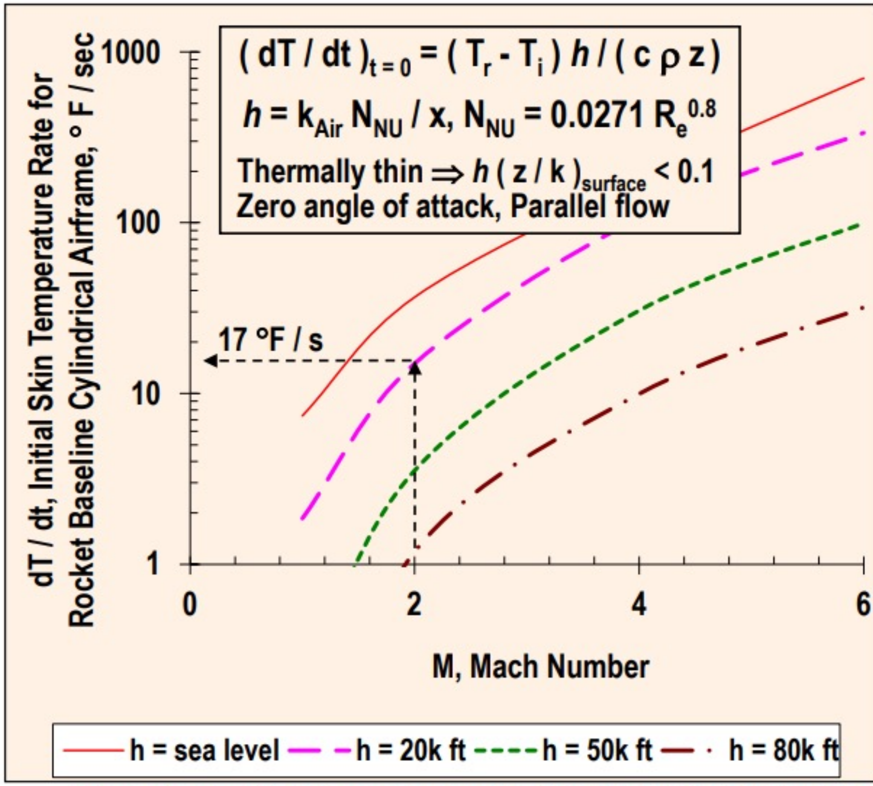


A Thermally Thin Surface → Higher heat Transfer
A Thermally Thick Surface → Lower Heat Transfer

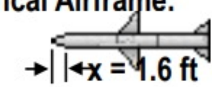


Nomenclature: T = Temperature, t = time, *h* = Convection heat transfer coefficient, z = Thickness ; k = Conductivity, α = Thermal Diffusivity

A "Thermally Thin" Surface in Aeero Heating has Rapid Temperature Rise



Example: Rocket Baseline Missile Cylindrical Airframe:



Aluminum skin w/o external insulation

$c = 0.215$ BTU / lbm / R, $\rho = 0.10$ lbm / in^3 = 172.8 lbm / ft^3, $z = 0.16$ in = 0.0133 ft, $k = 0.027$ BTU / s / ft / R / ft

Assume Mach 2 sustain, 20k ft altitude ($T_0 = 447$ R, $k_0 = 3.31 \times 10^{-6}$ BTU / s / ft / R, $\rho_0 = 0.001267$ slug / ft^3, $a_0 = 1037$ ft / s, $\mu_0 = 3.32 \times 10^{-7}$ slug / ft / s), Turbulent boundary layer, x = 1.6 ft

Calculate:

$Re_x = \rho_0\ M\ a_0\ x / \mu_0 = 12.7 \times 10^6$

$N_{NU} = 0.0271\ Re^{0.8} = 13000$

$h = k_0\ N_{NU} / x = 0.0268$ BTU / s / ft^2 / R

Test: $h\ (z/k)_{surface} = 0.0132 < 0.1$ ⇒ thermally thin

$T_r = T_0 [1 + 0.2\ r\ M^2] = 447 [1 + 0.2 (0.9)(2)^2] = 769$ R

At t = 0, assume $T_i = 460$ R = 0 °F ⇒

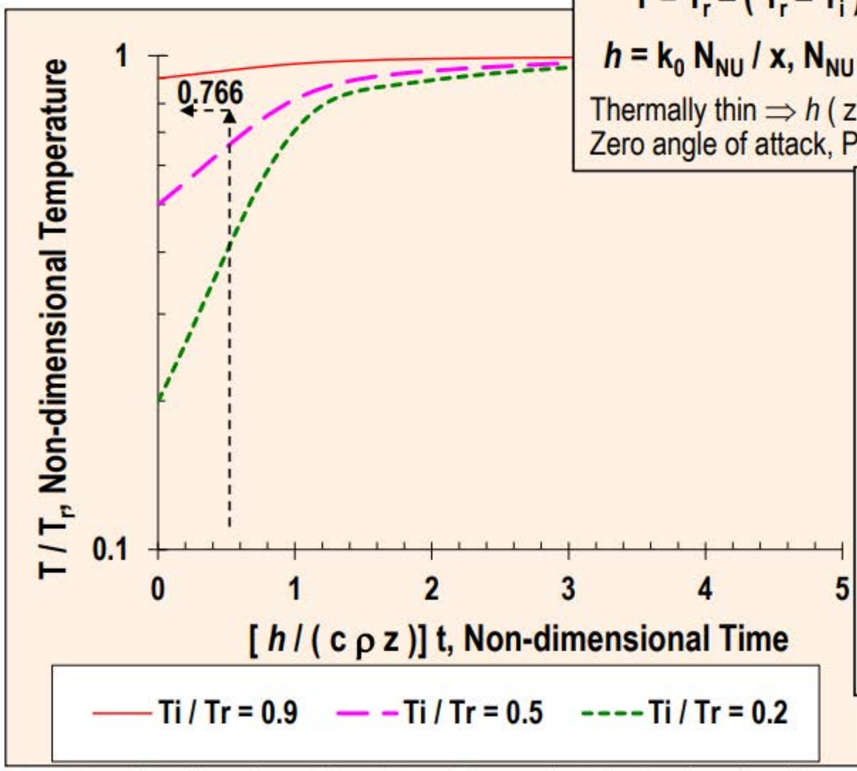
$(dT/dt)_{t=0} = (769 - 460)(0.0268) / [(0.215)(172.8)(0.01333)] = 17$ °F / s (9 °C / s)

Note: Aero heating; No external insulation; Thermally thin structure (uniform internal temperature); "Perfect" insulation behind airframe; 1-D heat transfer; Turbulent boundary layer; Radiation neglected

Nomenclature: dT / dt = Temperature rate, R / s; T_r = Recovery (max) temperature, R; T_i = Initial temperature, R; h = Convection heat transfer coefficient, BTU / s / ft^2 / R; c = Specific heat, BTU / lbm / R; ρ = Density, lb m / ft^3; z = Thickness, ft; k_{Air} = Conductivity of air, BTU / s / ft / R; Re = Reynolds number; N_{NU} = Nusselt number

Reference: Jerger, J.J., *Systems Preliminary Design Principles of Guided Missile Design*

A "Thermally Thin" Surface in Aero Heating has Rapid Temperature Rise (cont)

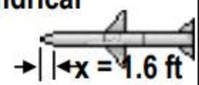


$$T = T_r - (T_r - T_i)\, e^{-ht/(c \rho z)}$$

$$h = k_0 N_{NU} / x, \quad N_{NU} = 0.0271\, R_e^{0.8}$$

Thermally thin $\Rightarrow h (z / k)_{surface} < 0.1$
Zero angle of attack, Parallel flow

Example: Rocket Baseline Missile Cylindrical Airframe:



Assume 10 s of Mach 2 sustain, 20k ft altitude, x = 1.6 ft

Using data from prior example

$T_i / T_r = 460 / 769 = 0.598$

$h t / (c \rho z) = 0.0268 (10) / [0.215 (172.8) (0.0133) = 0.542$

Compute

$T / T_r = 1 - (1 - T_i / T_r)\, e^{-ht/(c \rho z)} = 1 - (1 - 0.598) e^{-0.542} = 0.766$

$T = 0.766 (769) = 589\ R = 129\ °F (54\ °C)$

Note: Aero heating; No external insulation; Thermally thin structure (uniform internal temperature); "Perfect" insulation behind airframe; 1-D heat transfer; Turbulent boundary layer; Radiation neglected

Nomenclature: dT / dt = Temperature rate, R / s; T = Temperature, R; T_r = Recovery (max) temperature, R; T_i = Initial temperature, R; h = Convection heat transfer coefficient, BTU / s / ft^2 / R; c = Specific heat, BTU / lb / R; ρ = Density, lbm / ft^3; z = Thickness, ft; k_0 = Conductivity of air, BTU / s / ft / R; Re = Reynolds number; N_{NU} = Nusselt number

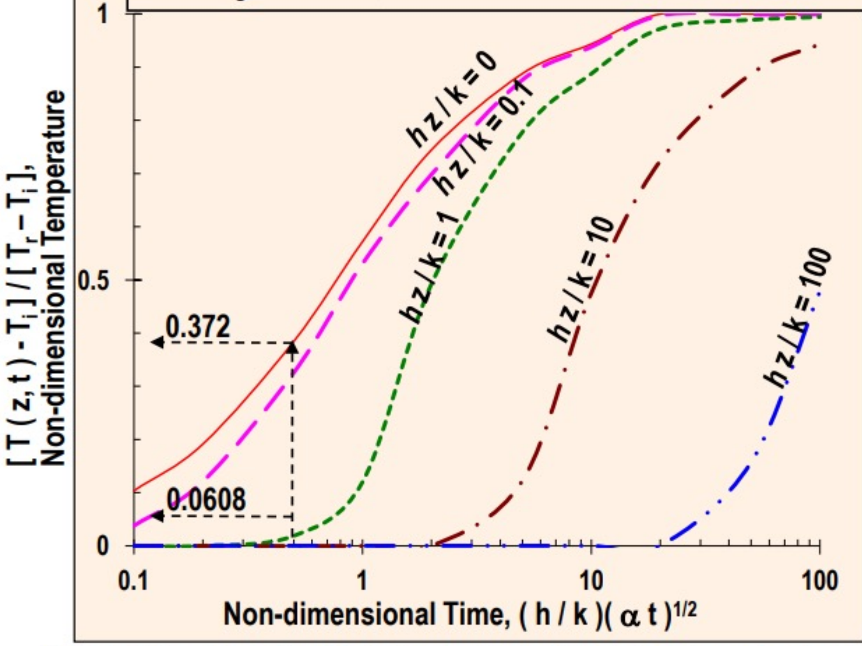
Reference: Jerger, J.J., *Systems Preliminary Design Principles of Guided Missile Design*

A “Thermally Thick” Surface in Aero Heating has Large Internal Temperature Gradient

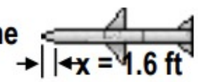
$$[T(z,t) - T_i]/[T_r - T_i] = \mathrm{erfc}\{z/[2(\alpha t)^{1/2}]\} - e^{(hz/k)+h^2\alpha t/k^2}\,\mathrm{erfc}\{z/[2(\alpha t)^{1/2}] + h(\alpha t)^{1/2}/k\}$$

$$[T(0,t) - T_i]/[T_r - T_i] = 1 - e^{h^2\alpha t/k^2}\,\mathrm{erfc}[h(\alpha t)^{1/2}/k]$$

Thermally thick surface ($z/[2(\alpha t)^{1/2}] > 1$), $T(z,0) = T_i$
Zero angle of attack, Parallel flow



Example: Rocket Baseline Missile Radome



$z = 0.25$ in $= 0.0208$ ft, $k = 5.96 \times 10^{-4}$ BTU / s / ft / R,
$\alpha = 1.499 \times 10^{-5}$ ft^2 / s
Mach 2, 20k ft alt ($T_0 = 447$ R), Turbulent boundary layer,
$x = 19.2$ in $= 1.6$ ft, $t = 10$ s, $T_r = 769$ R, $T_i = 460$ R
From Prior Example: $h = 0.0268$ BTU / s / ft^2 / R $\Rightarrow (h/k)(\alpha t)^{1/2} = 0.491$
Test: $z/[2(\alpha t)^{1/2}] = 0.0208/\{2[1.499\times10^{-5}(10)]^{1/2}\} = 0.849 < 1 \Rightarrow$ not quite thermally thick

Inner wall $\Rightarrow hz/k = 0.935$
$[T(0.0208, 10) - T_i]/[T_r - T_i] = 0.0608$
$T(0.0208, 10) = 479$ R (Note: $T_{inner} \approx T_i$)

Surface $\Rightarrow hz/k = 0$
$[T(0, 10) - T_i]/[T_r - T_i] = 0.372$
$T(0, 10) = 575$ R (319 K)

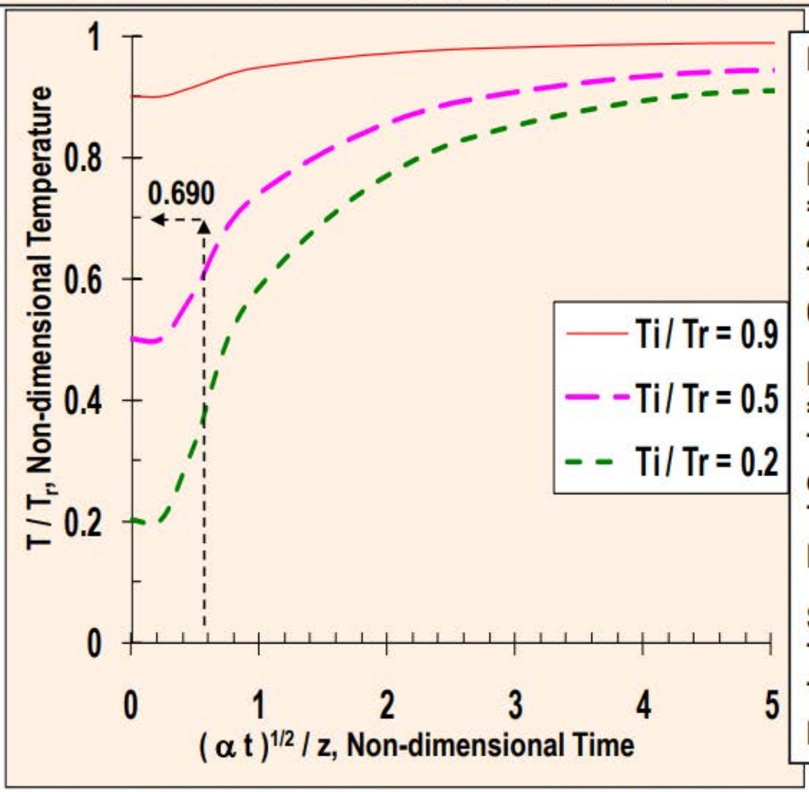
Note: 1-D heat transfer; Radiation neglected; Turbulent boundary layer
Nomenclature: T = Temperature, R; T_r = Recovery temperature, R; T_i = Initial temperature, R; h = Convection heat transfer coefficient, BTU / ft^2 / s / R; k = Thermal conductivity of material, BTU / s / ft / R; α = Thermal diffusivity of material, ft^2 / s; z_{max} = Thickness of material, ft; erfc = Complementary error function

Reference: Jerger, J.J., *Systems Preliminary Design Principles of Guided Missile Design*

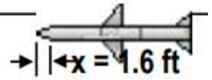
A "Thermally Thick" Surface in Aero Heating has Large Internal Temperature Gradient (cont)

Alternative (simpler) equation: $T(z, t) = T_r - (T_r - T_i)\,\mathrm{erf}\{ z / [2(\alpha t)^{1/2}]\}$

Assumptions: Thermally thick ($z / [2(\alpha t)^{1/2}] > 1$), $T(z, 0) = T_i$, $T(0, t) \approx T_r$



Example: Rocket Baseline Missile Radome



$z = 0.25$ in $= 0.0208$ ft , $\alpha = 1.499 \times 10^{-5}$ ft^2 / s
Mach 2, 20k ft alt ($T_0 = 447$ R), Turbulent boundary layer, x = 19.2 in = 1.6 ft, t = 10 s, $T_r = 769$ R, $T_i = 460$ R ⇨ $T_i / T_r = 460 / 769 = 0.598$
Test: $z / [2(\alpha t)^{1/2}] = 0.0208 / \{2[1.499\times10^{-5}(10)]^{1/2}\} = 0.849 < 1 \Rightarrow$ not quite thermally thick

Inner wall: $z = 0.0208$ ⇨ $(\alpha t)^{1/2} / z = 0.589$, $z / [2(\alpha t)^{1/2}]\} = 0.849$
$T / T_r = 1 - (1 - T_i / T_r)\,\mathrm{erf}[z / [2(\alpha t)^{1/2}]\} = 1 - (1 - 0.598)\,\mathrm{erf}\ 0.849 = 1 - 0.402(0.770) = 0.690 \Rightarrow$
$T = 0.690(769) = 531$ R
Note: Previous (more accurate) example gave T = 479 R

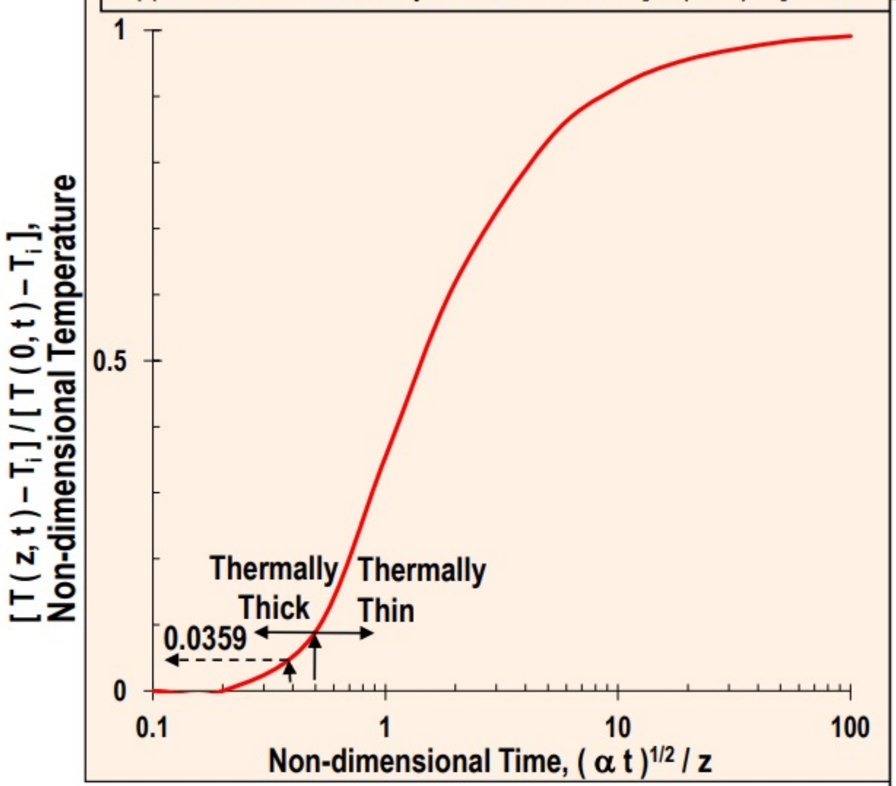
Surface: $z = 0$ ⇨ $(\alpha t)^{1/2} / z = \infty$, $z / [2(\alpha t)^{1/2}]\} = 0$
$T / T_r = 1 - (1 - 0.598)\,\mathrm{erf}\ 0 = 1 - 0.402(0) = 1 \Rightarrow$
$T = 1(769) = 769$ R (427 K)
Previous (more accurate) example gives T = 575 R (319 K)

Note: Zero angle of attack; Parallel flow; 1-D heat transfer; Radiation neglected; Turbulent boundary layer
Nomenclature: T = Temperature, R; T_r = Recovery temperature, R; T_i = Initial temperature, R; α = Thermal diffusivity of material, ft^2 / s; z_{max} = Thickness of material, ft; erf = Error function

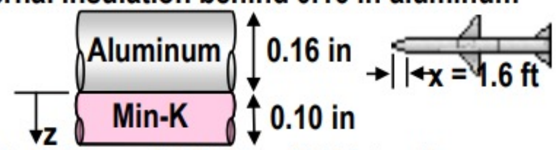
Airframe Internal Instulation Temperature can be Predicted Assuming Constant Flux Conduction

$$[T(z,t) - T_i] / [T(0,t) - T_i] = e^{-z^2/(4\alpha t)} - [\pi / (\alpha t)]^{1/2} (z/2)\, \text{erfc}\{ z / [2(\alpha t)^{1/2}]\}$$

Applicable for : Thermally thick surface: $z / [2(\alpha t)^{1/2}] > 1$, Zero angle of attack, Parallel flow



Example: Rocket Baseline Missile Internal Insulation
0.10 in Min-K internal insulation behind 0.16 in aluminum airframe skin



Assume M = 2, 20k ft alt, x = 1.6 ft, T_i = 460 R, t = 10 s, $z_{Min\text{-}K}$ = 0.10 in = 0.00833 ft, $\alpha_{Min\text{-}K}$ = 0.00000106 ft^2 / s, k = 5.96 x 10^{-4} BTU / s / ft, *h* = 0.0268 BTU / s / ft

Test: $z / [2(\alpha t)^{1/2}] = 0.00833 / \{2[0.00000106(10)]^{1/2}\}$ = 1.281 > 1 ⇒ thermally thick

Compute:
$(\alpha t)^{1/2} / z = [0.00000106(10)]^{1/2} / 0.00833 = 0.3907$
$[T_{Min\text{-}K}(0.0217, 10) - 460] / [T_{Min\text{-}K}(0, 10) - 460] = 0.0359$

Assume $(T_{inner})_{aluminum} = (T_{outer})_{Min\text{-}K}$
From prior example, $(T_{inner})_{aluminum}$ = 589 R at t = 10 s
Then, $(T_{outer})_{Min\text{-}K}$ = 589 R at t = 10 s

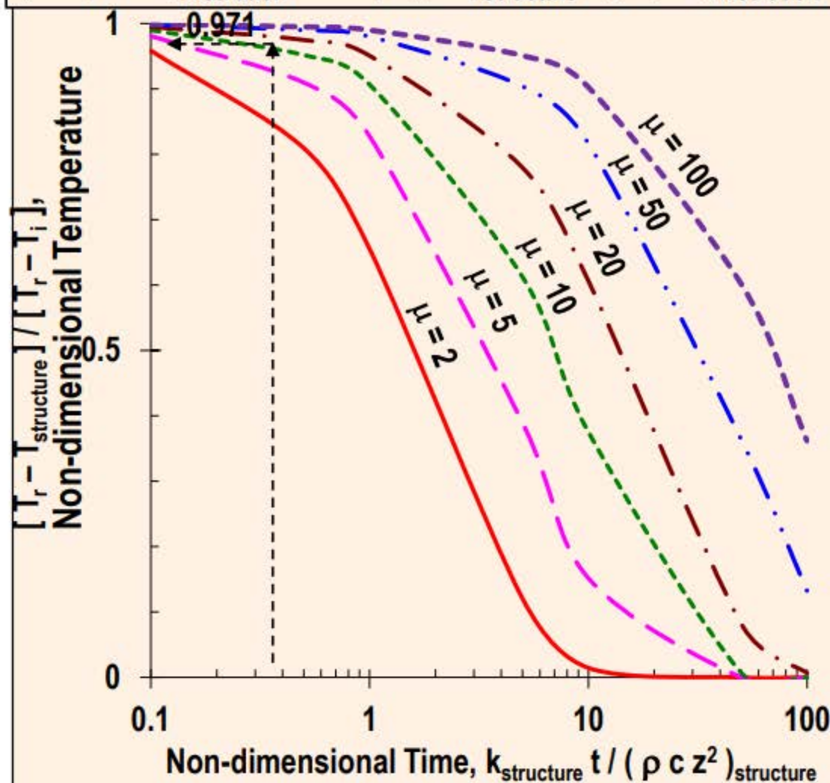
Compute, $(T_{inner})_{Min\text{-}K}$ = 460 + (589 – 460) 0.0338 = 460 + 4 = 464 R (249 K)

Note: 1-D conduction heat transfer; Radiation neglected; Constant heat flux input
Nomenclature: T (z,t) = Inner temperature of insulation at time t; T_i = Initial temperature; T (0, t) = Outer temperature of insulation at time t; α = Diffusivity of insulation material, ft^2 / s; z_{max} = Thickness of insulation material, ft; erfc = Complementary error function

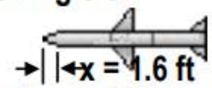
External Insulation Greatly Reduces Airframe Structure Temperature in Short Duration Flight

$$[\,T_r - T_{structure}\,] / [\,T_r - T_i\,] = e^{-[\,k_{insulation}\, t / (\rho c z^2)_{insulation}\,] / (0.406 + 0.981\,\mu)}$$

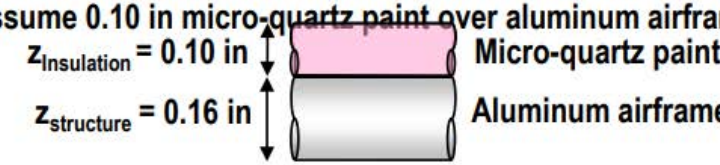
$$\mu = (k/z)_{Insulation} / h + (c \rho z)_{Structure} / (c \rho z)_{Insulation} + [(k/z)_{Insulation} / h]\,(c \rho z)_{Structure} / (c \rho z)_{Insulation}$$



Example for Rocket Baseline Missile at Zero Angle of Attack:



Assume 0.10 in micro-quartz paint over aluminum airframe



Assume M = 2, 20k ft alt, x = 1.6 ft, h = 0.0268 BTU / s / ft^2 / R, T_i = 460 R, t = 10 s

Micro-quartz paint insulation: k = 1.31 x 10^{-5} BTU / s / ft, c = 0.28 BTU / lbm / °F, ρ = 0.012 lbm / in^3 = 20.74 lbm / ft^3

Aluminum airframe: k = 0.027 BTU / s / ft / °F, c = 0.22 BTU / lbm / °F, ρ = 0.103 lbm / in^3

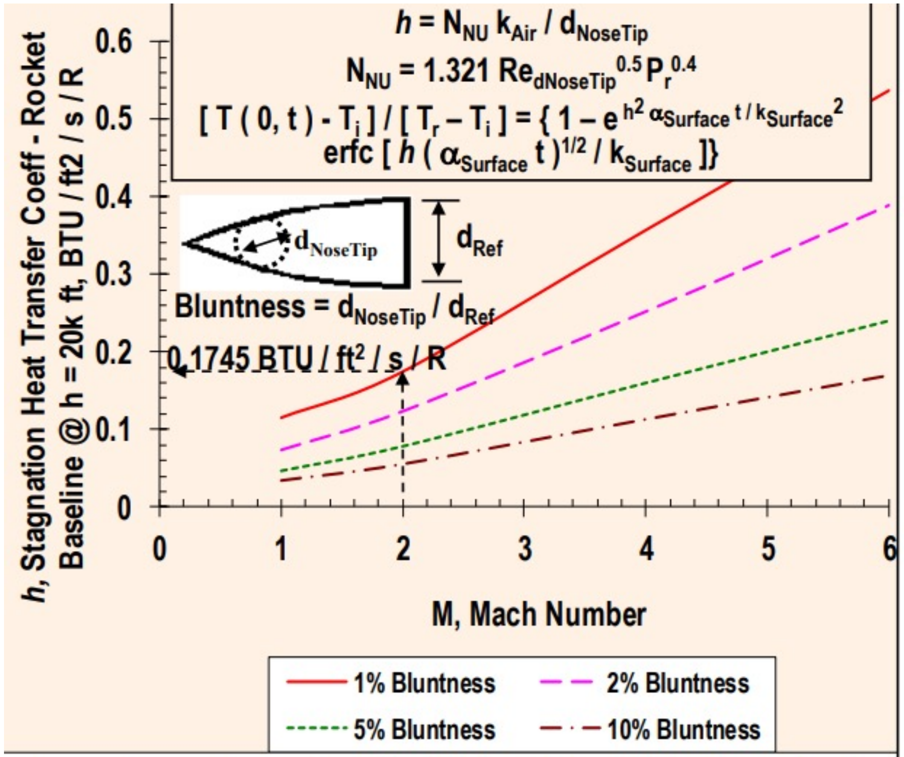
Compute:

$k_{insulation}\, t / (\rho c z^2)_{insulation}$ = 1.31 x 10^{-5} (10) / [(20.74)(0.28)(0.00833)^2] = 0.325, μ = 0.000587 + 10.91 + 0.0064 = 10.92

$[\,T_{structure} - T_i\,] / [\,T_r - T_i\,] = e^{-0.325 / [(0.406 + 0.981\,(10.92)]} = 0.971$ ⇨ $T_{structure}$ = 469 R (261 K)

Note: 1-D conduction; Radiation neglected; Constant heat input; Temp constant through structure; "Perfect" insulation behind structure; Nomenclature: $T_{structure}$ = Temp of structure, R; T_i = Initial temp, R; T_r = Recovery temp, R; t = Time, s, z = Thickness, ft; k = Thermal conductivity, BTU / s / ft / R; c = Specific heat, BTU / lbm / R; ρ = Density, lbm / ft^3; μ = Aggregate thermal resistance coefficient

A Sharp Nose Tip / Leading Edge has High Aerodynamic Heating in Hypersonic Flight



Example: Rocket Baseline Missile Nose Tip:

Assume: 1% bluntness, M = 2 @ h = 20k ft (T_0 = 447 R, ρ_0 = 0.001267 slug / ft^3) $\Rightarrow$ T_T = Recovery Temperature = T_0 (1 + 0.16 M^2) = 733 R (laminar boundary layer heating)

Calculate: $d_{NoseTip} / d_{Ref} = 0.01 \Rightarrow d_{NoseTip} = 0.01\,(8) = 0.08$ in = 0.00667 ft

$\mu_{Air} = 1.147\times10^{-5}\,[\,708 / (T + 216\,](T / 492)^{3/2}\,] = 1.665 \times 10^{-5}$ lbm / s / ft

$Re_{d,NoseTip} = \rho_0 V_0 d_{NoseTip} / \mu = 3.39 \times 10^4$

$k_{Air} = 3.58\times10^{-6}\,[\,717 / (T_T + 225)](T_T / 492)^{3/2} = 5.22\times10^{-6}$ BTU / s / ft / R

$c_p = 0.122\, T^{0.109} = 0.122\,(805)^{0.109} = 0.253$ BTU / lbm / R

$P_r = c_p\, \mu / k = 0.253\,(1.665 \times 10^{-5}) / 5.22\times10^{-6} = 0.807$

$N_{NU} = 1.321\,(3.39 \times 10^4)^{0.5}\,(0.807)^{0.4} = 223$

$h = 223\,(5.22\times10^{-6}) / 0.00667 =$ **0.1745 BTU / ft^2 / s / R**

Calculate radome tip outer temperature after 10 s sustain:

$\alpha_{Surface}$ = Thermal diffusivity of radome = 1.499×10^{-5} ft^2 / s

$k_{Surface}$ = 5.96×10^{-4} BTU / s / ft / R

$[\,T(0, 10) - 460\,] / [\,733 - 460\,] = 1 - e^{[(0.1745)^2 (1.499 \times 10^{-5})(10) / (5.96 \times 10^{-4})^2]}\ \mathrm{erfc}\,\{ (0.1745)\,[\,1.499 \times 10^{-5}\,(10)]^{1/2} / (5.96 \times 10^{-4})]\} \approx 1$

T (0, 10) = 460 + 273 (1) = 733 R (407 K) $\approx$ recovery temp

Assumptions: Zero angle of attack; Parallel flow; 1-D heat transfer; Laminar boundary layer

Note: Nose tip / leading edge bluntness requires consideration of drag, localized stress, seeker performance, and aero heating

Nomenclature: *h* = Convection heat transfer coefficient for stagnation recovery, BTU / s / ft^2 / R; N_{NU} = Nusselt number for stagnation recovery; k = Air thermal conductivity at stagnation recovery, BTU / s / ft / R; c_p = specific heat at constant pressure, BTU / lbm / R; $d_{NoseTip}$ = Nose tip diameter, ft; $Re_{dNoseTip}$ = Reynolds number based on nose tip diameter; P_r = Prandtl number; μ_{Air} = Viscosity of air; α = Air thermal diffusivity

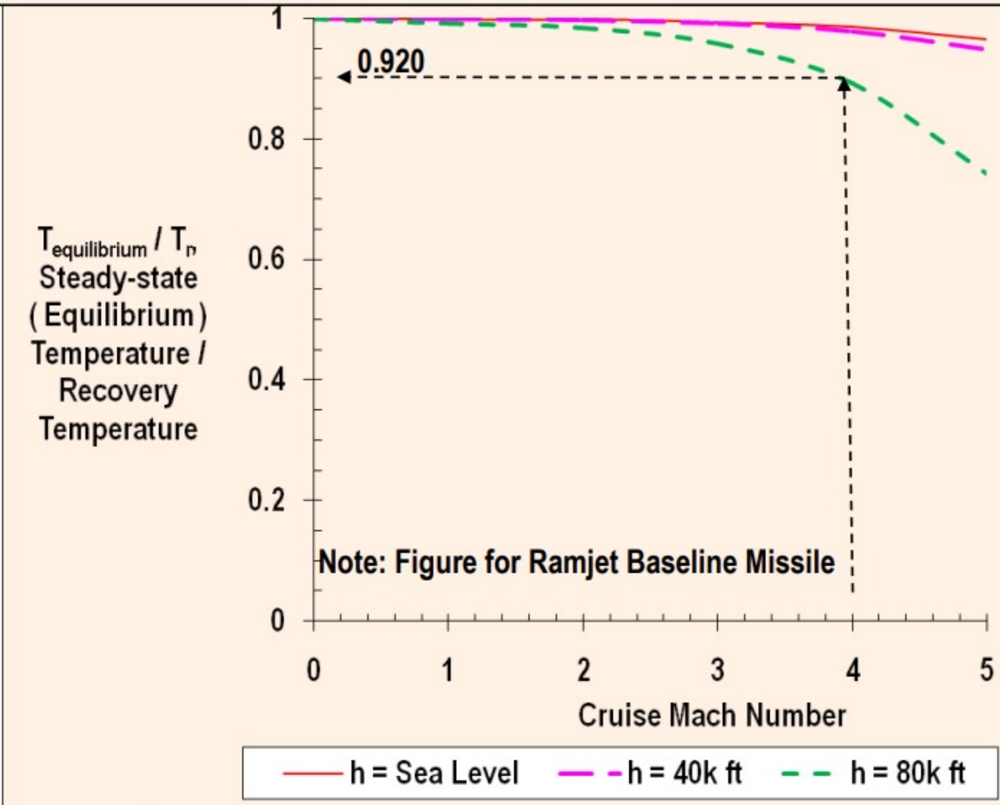
Reference: Allen, J. and Eggers, A. J., “NACA Report 1381”

Missiles Have Relatively Low Radiation Heat Loss at Moderate Temperature / Mach Number

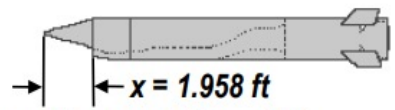
$Q_{input} = Q_{output}$

$Q_{input} = h\,(T_r - T_{equilibrium})$, $Q_{output} = Q_{rad} = 4.76 \times 10^{-13}\,\varepsilon\,T^4$, BTU / ft² / s

Applicable for Long Duration (Steady-state, Equilibrium) Cruise Flight



Example: $T_{equilibrium}$ for Ramjet Baseline Missile



- Titanium skin with emissivity $\varepsilon = 0.3$
- Assume x = 1.958 ft (behind radome)
- Assume long duration (equilibrium) heating at Mach 4
- Assume altitude h = 80k ft $\Rightarrow T_0 = 398$ R, $\rho_0 = 0.000084$ slug / ft³, $\mu_0 = 3.02 \times 10^{-7}$ = lbm / s / ft
- Assume turbulent boundary layer (r = 0.9)

Compute

- $T_r = T_0(1 + 0.2rM^2) = 398[1 + 0.2(0.9)(4)^2] = 1544$ R
- $R_e = \rho V x / \mu = 0.000084\,(978)\,(1.958) / 3.02 \times 10^{-7} = 2.17 \times 10^6$
- $h = k\,N_{NU} / x = 0.00487$ BTU / ft² / s / R
- $Q_{output} = Q_{rad} = 4.76 \times 10^{-13}\,(0.3)\,T_{equilibrium}^4 = 1.428 \times 10^{-13}\,T_{equilibrium}^4$
- $Q_{input} = h\,(T_r - T_{equilibrium}) = 0.00487\,(1544 - T_{equilibrium})$

Setting $Q_{input} = Q_{output} = Q_{rad}$ and solving for $T_{equilibrium}$

$T_{equilibrium} = 1377$ R $\Rightarrow T_{equilibrium} / T_r = 0.920$

Note: Q_{rad} = Radiation heat flux, h = Convection heat transfer coefficient, T_r = Recovery temperature, T_0 = Free stream temperature, r = Recovery factor, R_e = Reynolds number, ρ = Atmospheric density, μ = Atmospheric viscosity

A Missile Design Concern is Localized Aerodynamic Heating and Thermal Stress

IRdome / Radome

- Large temp gradients due to low thermal conduction
- Thermal stress at attachment
- Low tensile strength
- Dome fails in tension

Sharp Leading Edge / Nose Tip

- Hot stagnation temperature on leading edge
- Small radius prevents use of external insulation
- Cold heat sink material as chord increases in thickness leads to leading edge warp
- Shock wave interaction with adjacent body structure

Body Joint

- Hot missile shell
- Cold frames or bulkheads
- Causes premature buckling

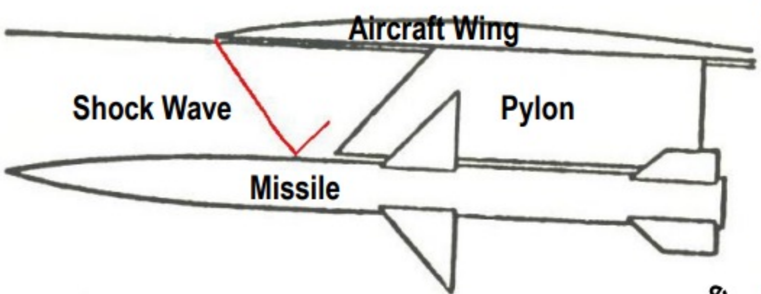
Note: σ_{TS} = Thermal stress from restraint in compression or tension = α E ΔT
Nomenclature: α = coefficient of thermal expansion, E = modulus of elasticity, $\Delta T = T_2 - T_1$ = temperature difference

**Example: Thermal Stress σ_{TS} for Rocket Baseline Missile Pyroceram Dome ($\alpha = 3 \times 10^{-6}$ / R, E = 13.3×10^6 psi, σ_{max} = 25,000 psi)
Assume M = 2, h = 20k ft altitude, t = 10 s. From prior figures: $\Delta T = T_{OuterWall} - T_{InnerWall}$ = 575 – 479 = 96 R (Jerger Reference), ΔT = 769 – 531 = 238 R (Carslaw and Jaeger Reference)
Then $\sigma_{TS} = 3 \times 10^{-6} (13.3 \times 10^6) (96)$ = 3830 psi (Jerger), $\sigma_{TS} = 3 \times 10^{-6} (13.3 \times 10^6) (238)$ = 9500 psi (Carslaw and Jaeger)**

Note: Carslaw and Jaeger Reference Less Accurate Because of Approximation $T (0, t) \approx T_r$

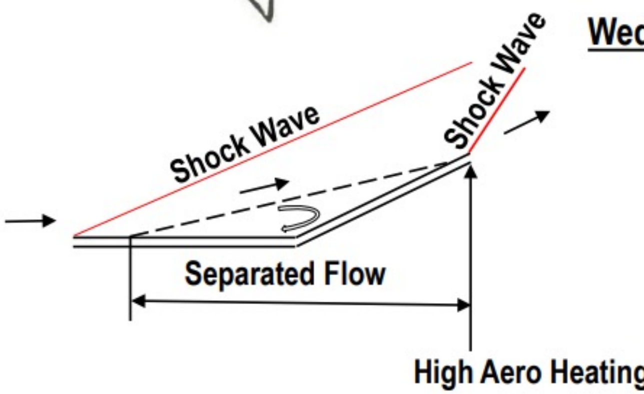
A Missile Design Concern is Localized Aerodynamic Heating and Thermal Stress (cont)

Shock Wave – Boundary Layer Interaction



Shock Wave from Surface Leading Edge Intercepts Another Surface

- High localized heating
- Example: shock wave from aircraft wing intercepts store

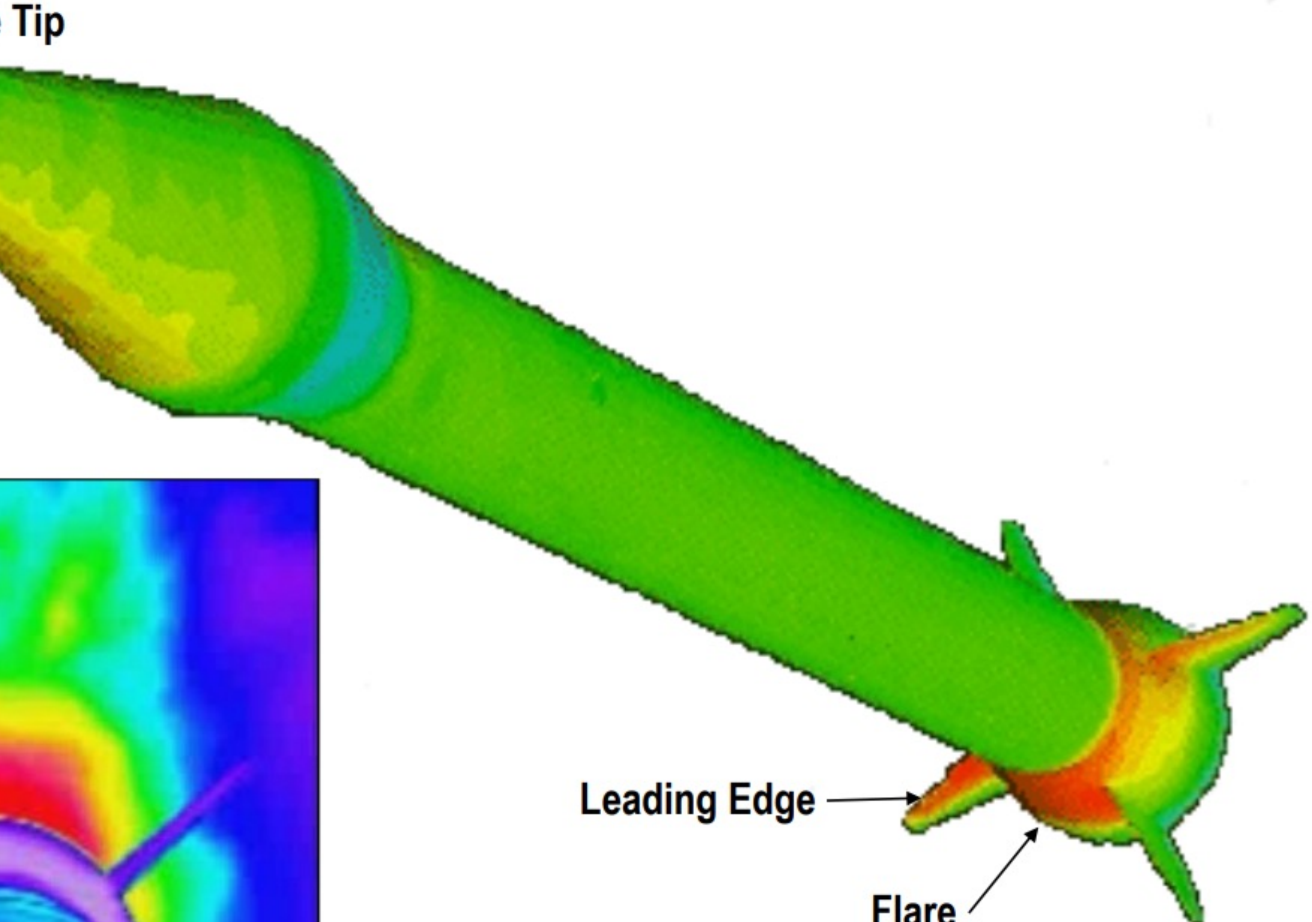


Wedge / Flare Corner Flow

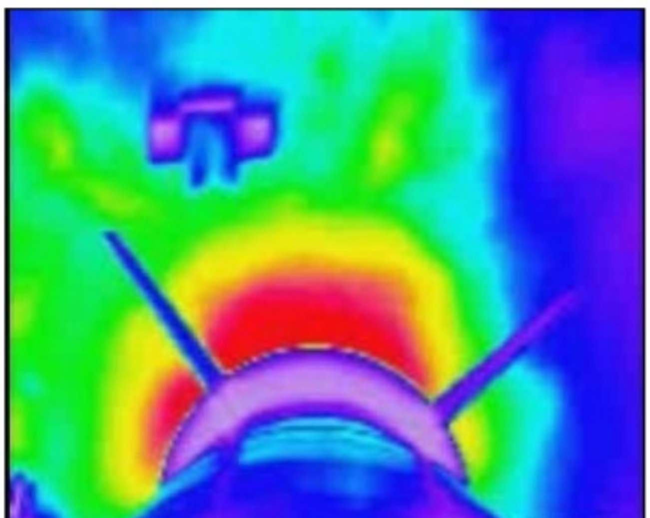
- Separated flow at corner
- Shock wave at reattachment
- High heating at reattachment (r ≈ 1)

Video: 24 Deg Flare at Mach 2.3

Examples of Aerodynamic Hot Spots

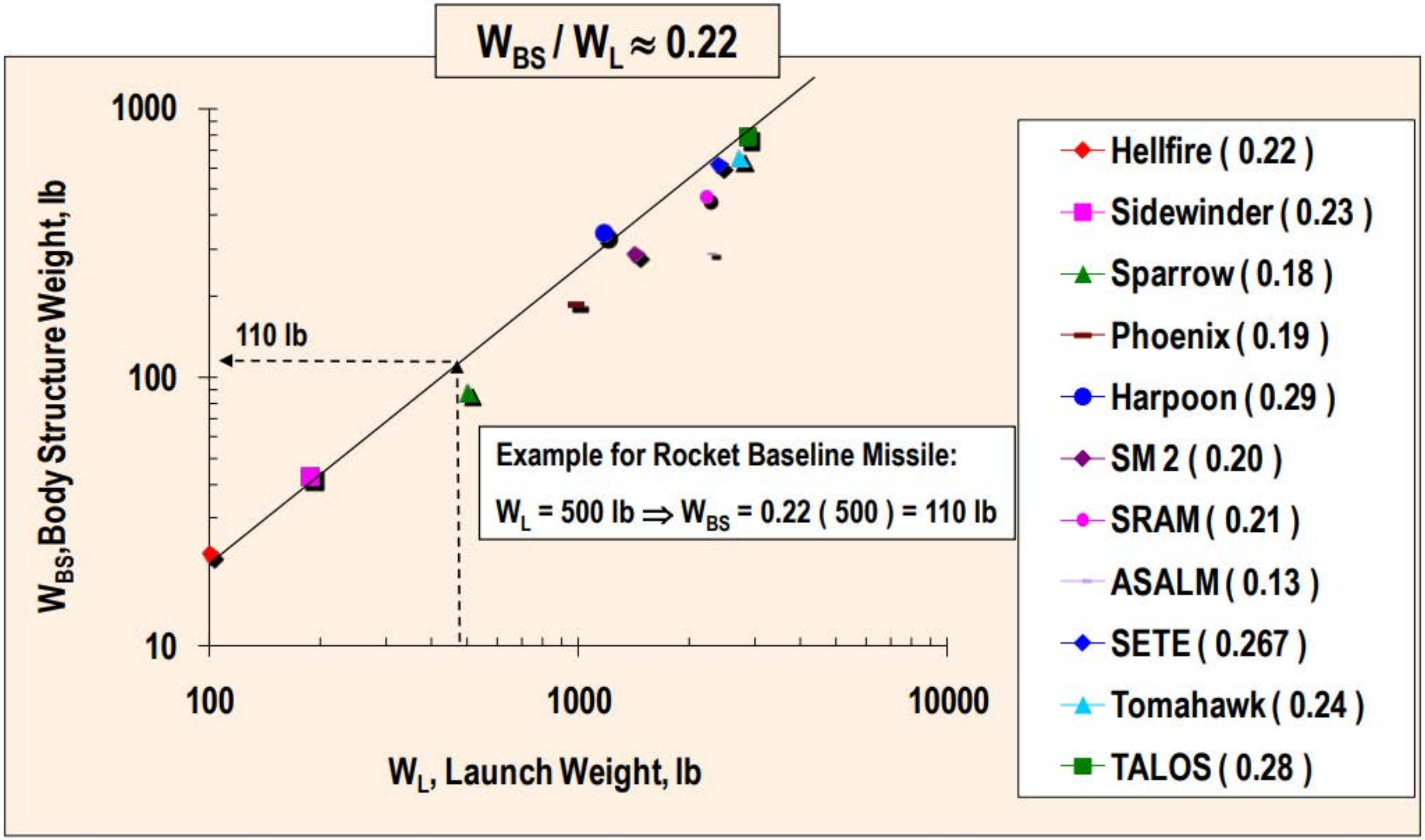


Notional Missile Aero Heating



Video of Radiometric Imagery – SM-3 Flight

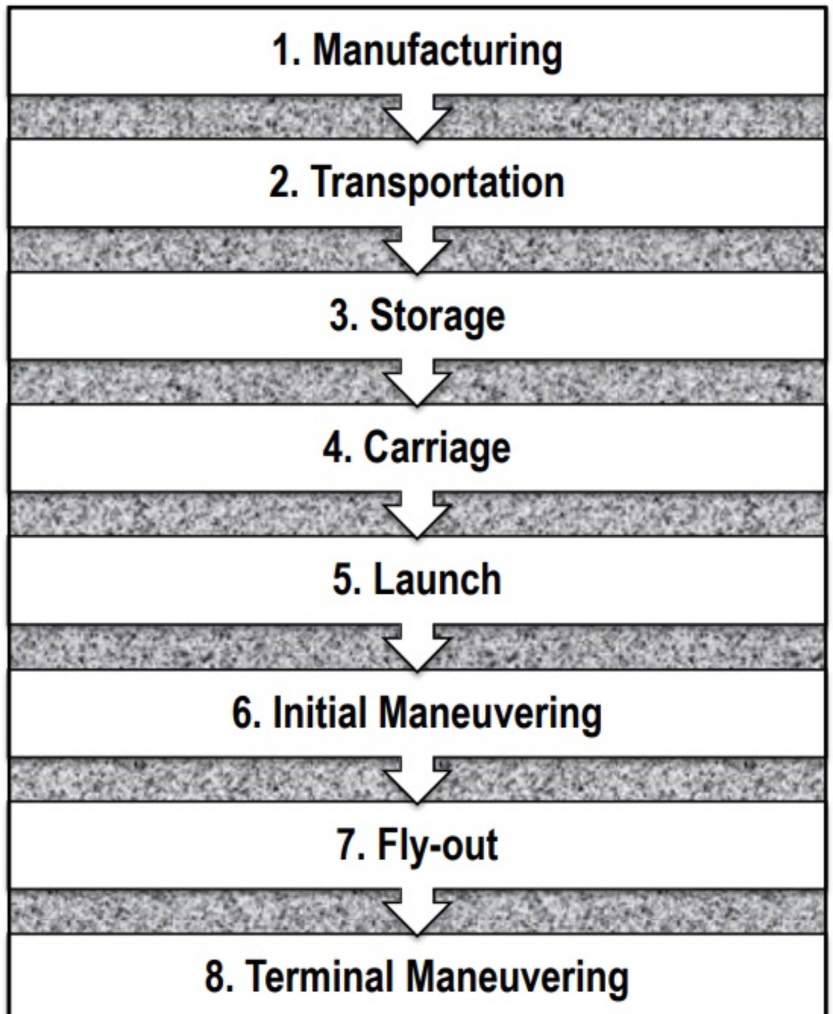
Missile Metal Body Structure Weight is about 22% of the Missile Launch Weight



Note: W_{BS} includes all load carrying body structure. If motor case, engine, or warhead case carry external loads then they are included in W_{BS}. W_{BS} does not include tail, wing, or other surface weight.

Note: Above based on metal structure. Graphite composite structure would result in lower body structure weight fraction.

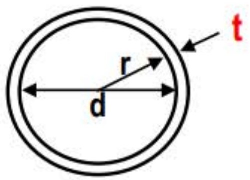
Missile Structure is Based on Considering the Cradle-to-Grave Environment



Missile Body Structure Required Thickness is Based on Considering Many Design Conditions

Contributors to Body Structure Thickness	*Cylindrical Body Structure Thickness Equation*
Min Gauge for Manufacturing	$t = 0.7\ d\ [(\ p_{ext} / E\ )\ l / d\ ]^{0.4}$, $t \approx 0.06$ in, if $p_{ext} \approx 10$ psi
Localized Buckling in Bending	$t = 2.9\ r\ \sigma / E$
Localized Buckling in Axial Compression	$t = 4.0\ r\ \sigma / E$
Thrust Force	$t = T / (\ 2\ \pi\ \sigma\ r\ )$
Maneuver Bending Moment	$t = M / (\ \pi\ \sigma\ r^2\ )$
Internal Pressure	$t = p\ r / \sigma$

Reference: Atkinson, J.R. and Staton, R.N., "Missile Body Weight Prediction", SAWE 1497, May 1982



Note: Does not include factor of safety (FOS)

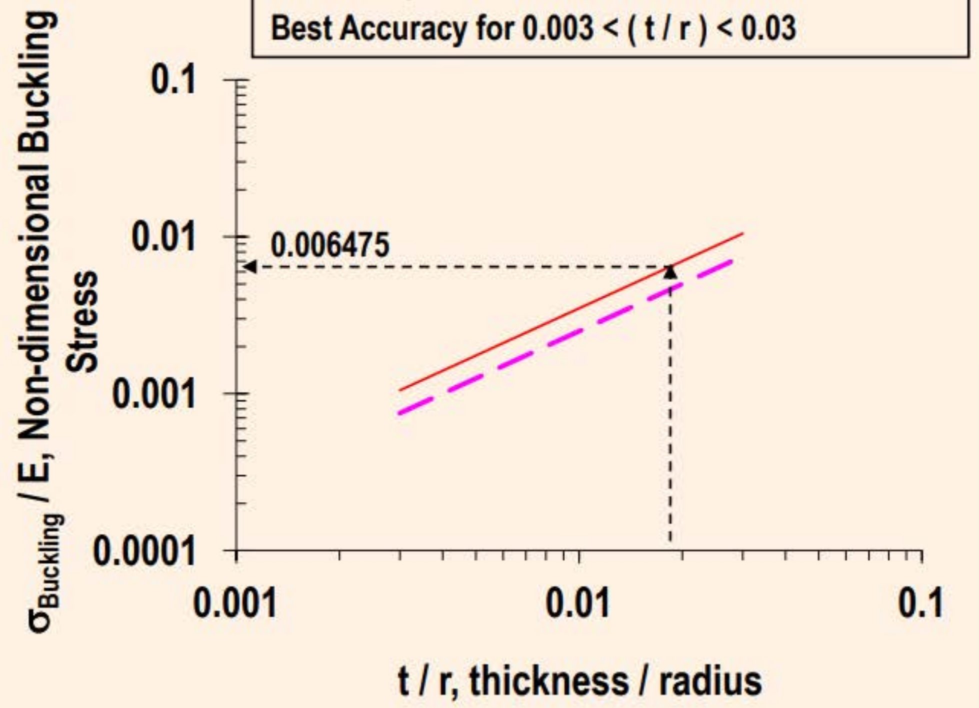
σ = strength, E = modulus of elasticity

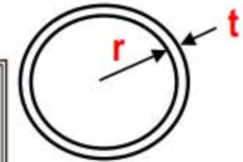
Localized Buckling May be Concern for a Thin Wall Structure

$\sigma_{Buckling,Bending} / E \approx 0.35 (t / r)$

$\sigma_{Buckling,AxialCcompression} / E \approx 0.25 (t / r)$

Best Accuracy for $0.003 < (t / r) < 0.03$

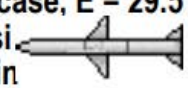




Note: Thin wall cylinder with local buckling
$\sigma_{Buckling} / E$ = Non-dimensional buckling stress
$\sigma_{BucklingBending}$ = Buckling stress in bending
$\sigma_{BucklingAxialCompression}$ = Buckling stress in axial compression
E = Young's modulus of elasticity
t = Airframe thickness
r = Airframe radius
Min thickness for fab and handling $\approx$ 0.06 in

Example: Rocket Baseline Missile Buckling in Bending:

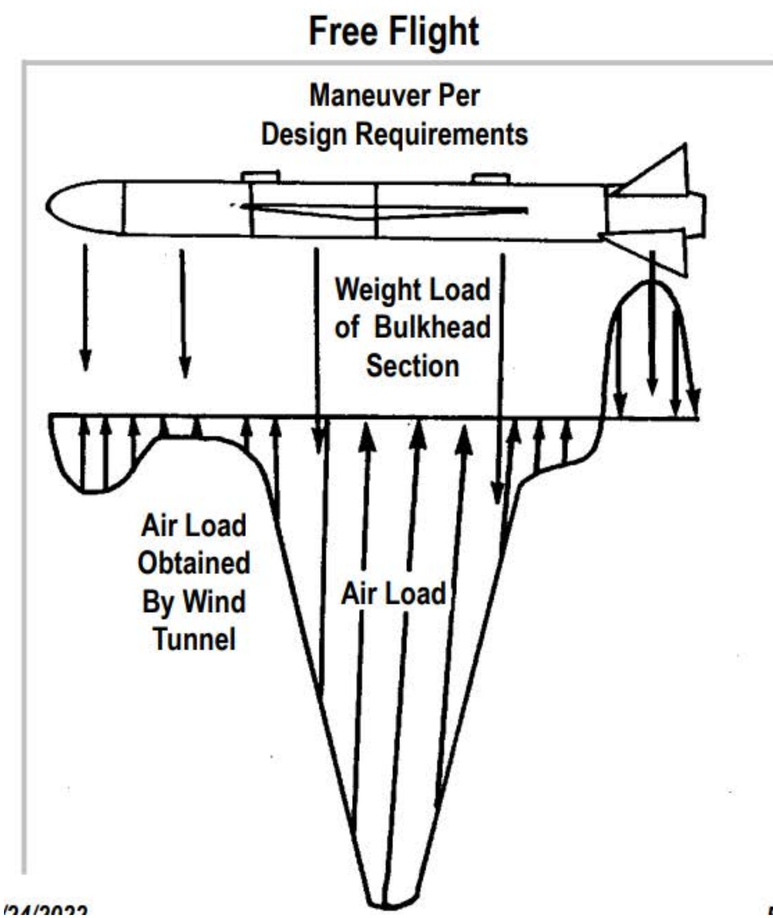
4130 steel motor case, $E = 29.5 \times 10^6$ psi
σ_{yield} = 170,000 psi
t = 0.074 in, r = 4 in
t / r = 0.0185
$\sigma_{Buckling,Bending} / E \approx 0.35 (0.0185) = 0.006475$
$\sigma_{buckling,Bending} \approx$ 191,000 psi (1.37×10^9 Pascal)
$\sigma_{buckling,Bending} \sim \sigma_{yield}$

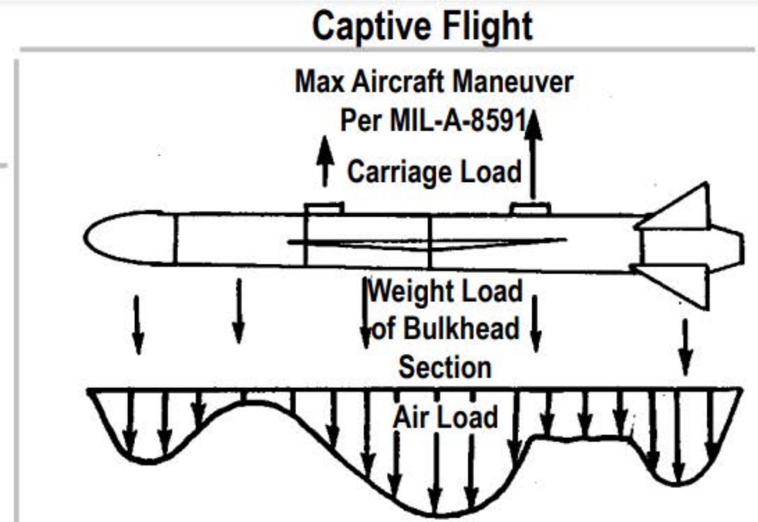


Note: Actual compression buckling stress can vary +/- 50%, depending upon typical imperfections in geometry and loading symmetry.

Note: Best type of steel for motor case is a tradeoff of strength, hardness (machining), buckling, joining / welding, and cost.

MIL-A-8591 Provides a Conceptual Design Procedure to Estimate Captive Carriage Max Load





Note: MIL-A-8591 Procedure A assumes a worst case where the max air loads flight condition combines with the max g forces flight condition, regardless of different angles of attack (α).

Example of Max Angle of Attack α_{max} Calculated by MIL-A-8591 Using Procedure A for F-18 Aircraft Carriage at max maneuver g ($n_{z,max}$) :

$\alpha_{max} = 1.5\ [\ n_{z,max}\ W_{max} / (\ C_{L_\alpha}\ q\ S_{Ref}\ )]_{aircraft}$

$\alpha_{max} = 1.5\ (\ 7.5\ )\ (\ 49200\ ) / [\ 0.05\ (\ 1481\ )\ (\ 400\ )] =$ 18.7 deg

Maximum Body Bending Moment Depends Upon Load Distribution

Example for Rocket Baseline Missile Body: 

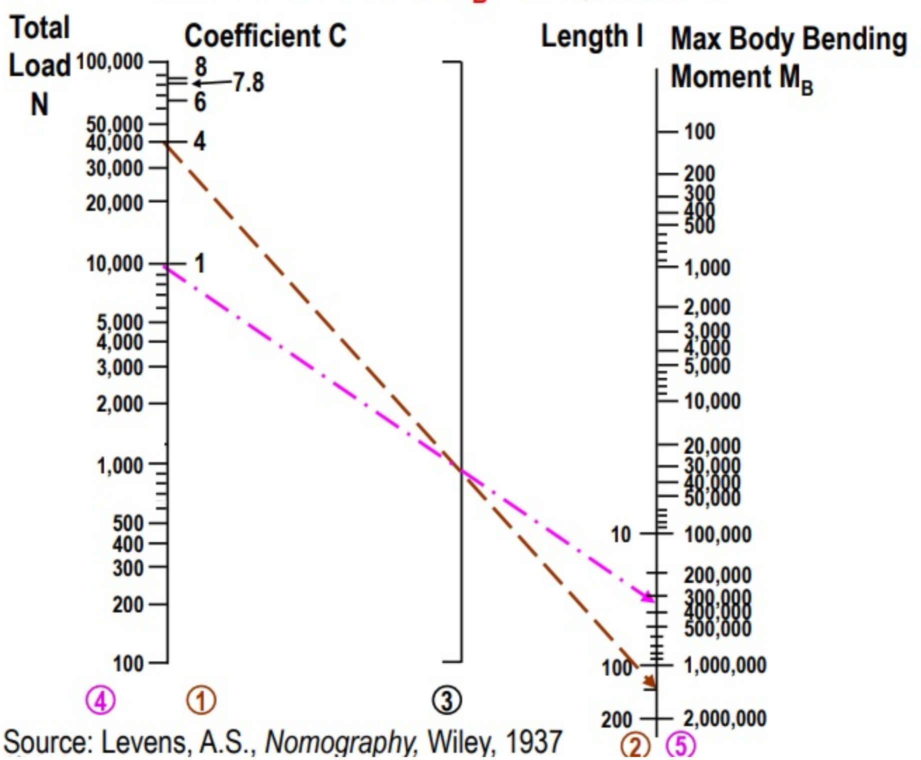
① c = 4, center normal load

② l = 144 in

① to ② ⇒ — — ③

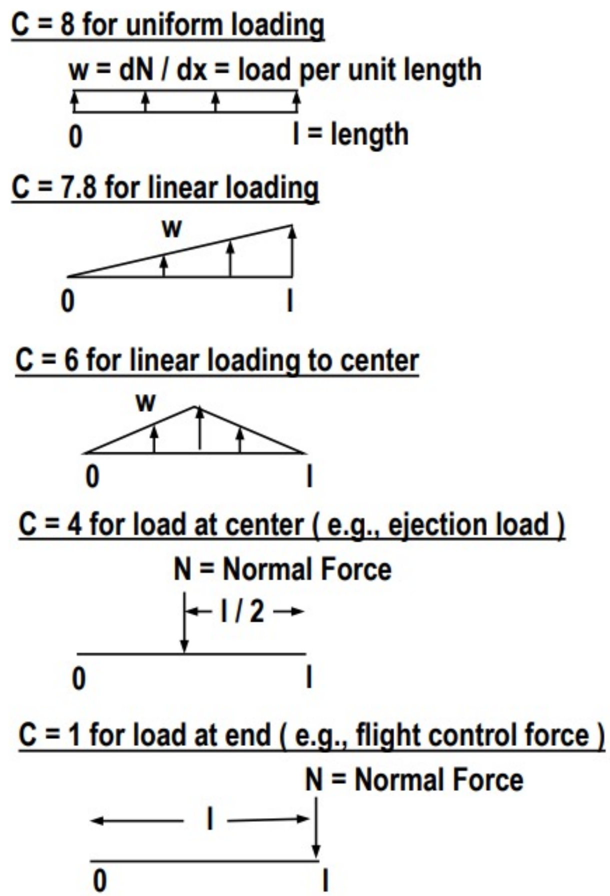
④ N = 10,000 lb (20 g)

④ to ③ ⇒ —·— ⑤ M_B = 360,000 in-lb

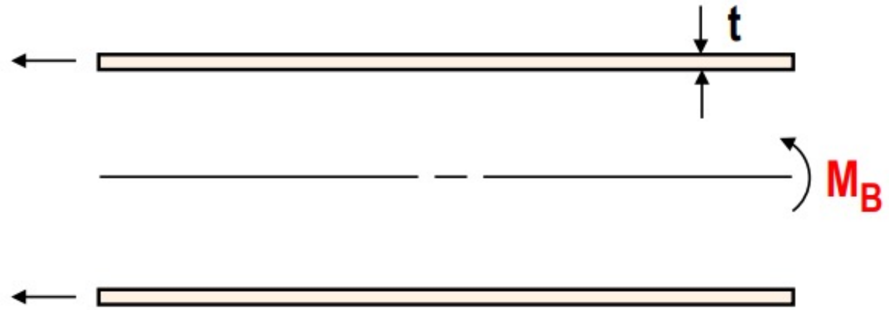


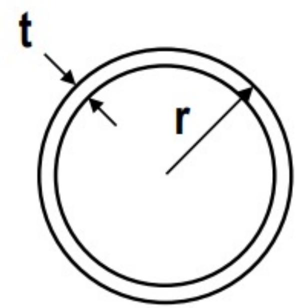
Source: Levens, A.S., *Nomography*, Wiley, 1937

$$M_B = N\,l / c$$



Body Bending Moment May Drive Body Structure Thickness / Weight





$A = 2 \pi r t$

$I_z = I_Y = \pi r^3 t$

$$t = M_B (FOS) / [\pi r^2 \sigma_{max}]$$

Example for Rocket Baseline Missile Body Structure:

- **Body has circular cross section**
- **2219-T81 aluminum skin ($\sigma_{ultimate,tensile}$ = 65,000 psi)**
- **$r = 4$ in**
- **Normal ejection load = 10,000 lb (20 g)**
- **Bending moment M_B = 360,000 in · lb**
- **Factor of safety FOS = 1.5**

$t = 360{,}000 (1.5) / [\pi (4)^2 (65{,}000)] = 0.16$ in

Note / Assumptions:

Thin cylinder

Circular cross section

Solid skin

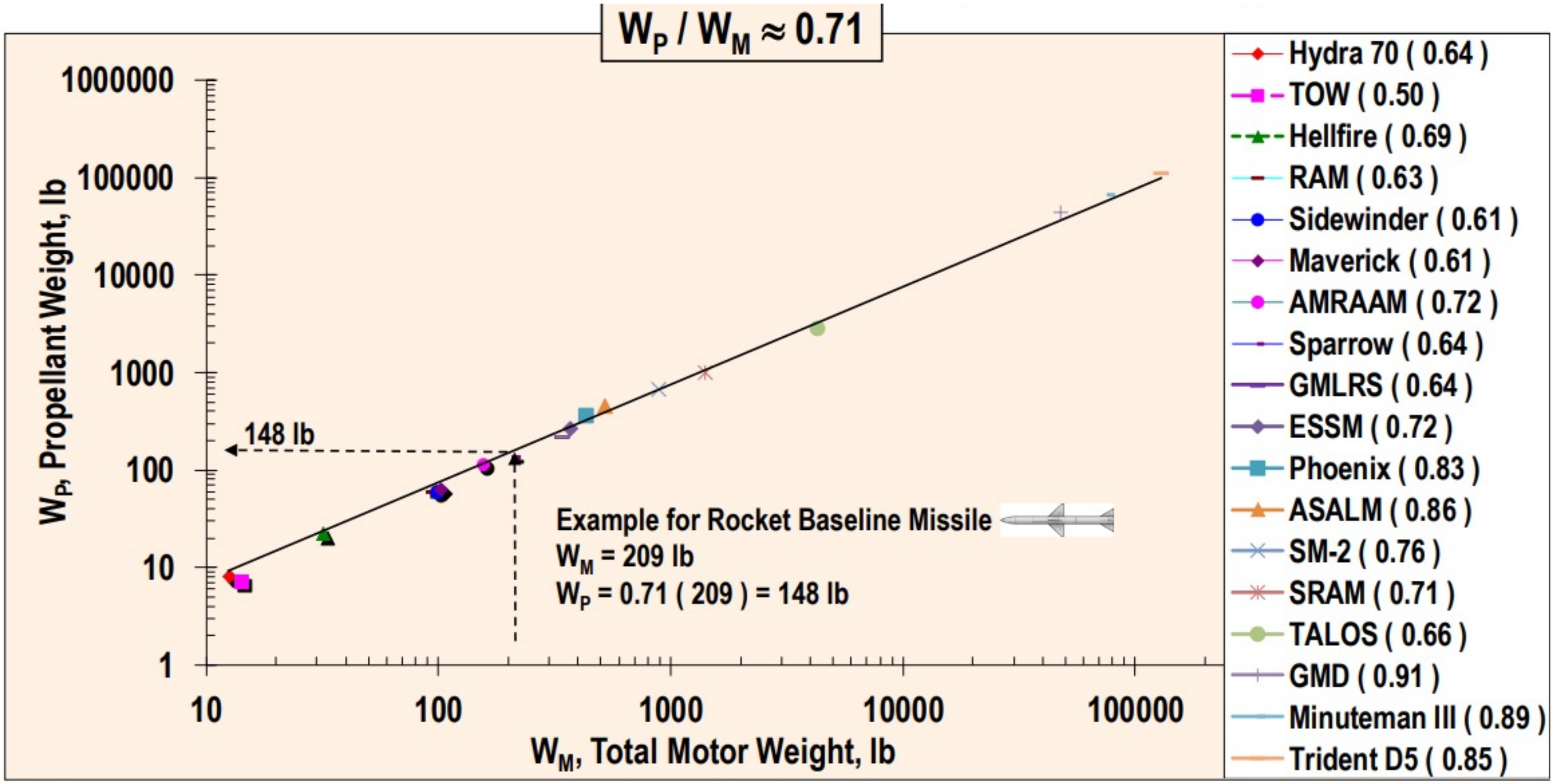
Longitudinal strength

Axial load stress and thermal stress assumed small compared to bending moment stress

$\sigma = M_B r / I_Z = M_B r / (\pi r^3 t)$

$= M_B / (\pi r^2 t)$

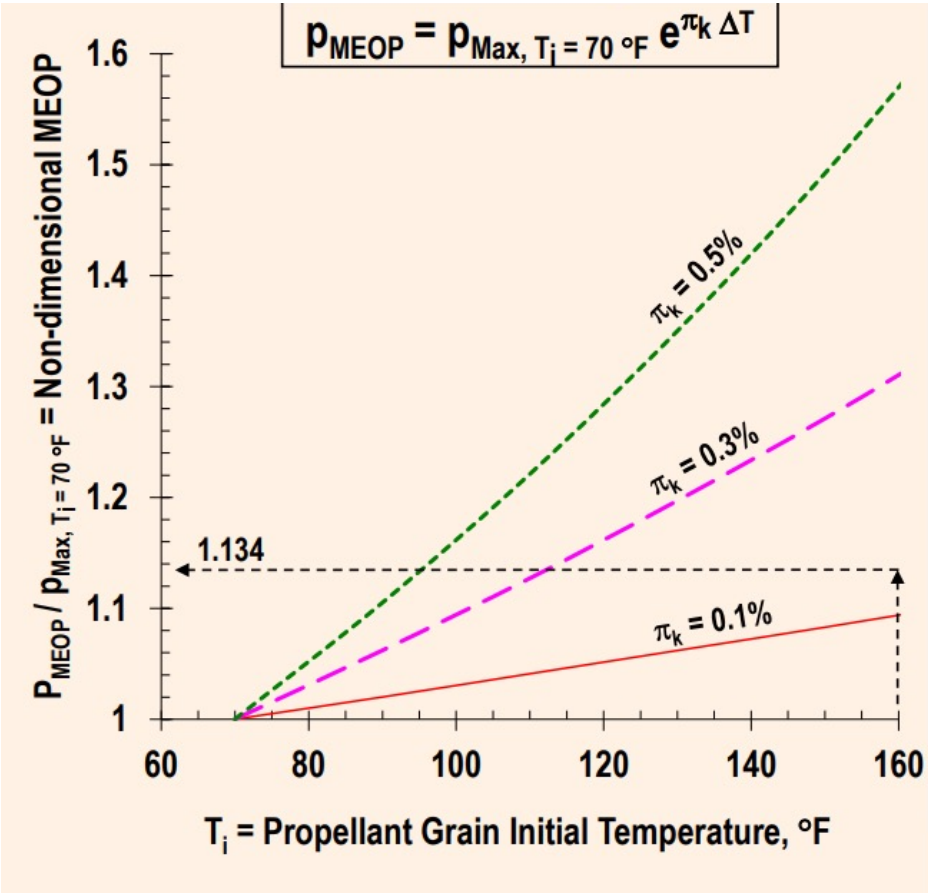
For a Typical Solid Propellant Rocket Motor, about 71% of the Motor Weight is Propellant



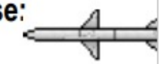
Note:

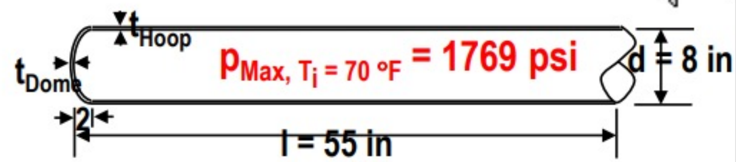
- Correlation based on solid propellant rocket
- W_M includes propellant, motor case, nozzle, and insulation.
- Drivers include volumetric loading, motor case strength, motor case density, chamber pressure, flight loads, and burn time,

Maximum Expected Operating Pressure (MEOP) Increases with Propellant Grain Initial Temperature



Example: Rocket Baseline Missile Motor Case:





Calculate p_{MEOP} for $\pi_k = 0.14\%$, $T_i = 160\ °F$

$p_{MEOP} / p_{Max,\ T_i = 70\ °F} = e^{0.0014\,(160 - 70)} = 1.134$

$P_{MEOP} = 1.134\ (1769) = 2007$ psi or 13.84×10^6 Pascals

Note: Conceptual Design Burst Pressure p_{Burst} Is Greater Than Nominal P_{MEOP}, Due to Uncertainty in Ignition Spikes, Welds, etc.

Assume p_{Burst} Uncertainty Can Be Modeled As Standard Deviation $\sigma = p_{MEOP} / p_{Max,\ T_i = 70\ °F} = 2007 / 1769 = 1.134$

Assume 2σ Uncertainty in p_{Burst}

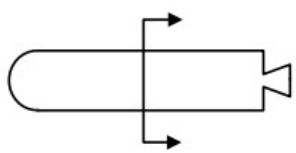
Calculate

$p_{Burst} \approx 2007\ (1.134)^2 = 2582$ psi or 17.80×10^6 Pascals

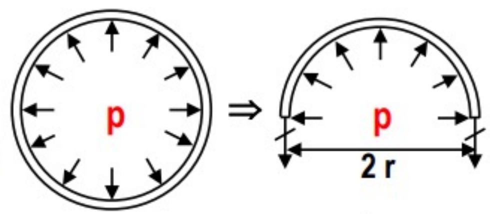
Nomenclature: p_{MEOP} = Maximum Expected Operating Pressure, $p_{Max,\ T_i = 70\ °F}$ = Maximum Pressure for Propellant Grain Initial Temperature = 70 °F, π_k = Coefficient of Pressure Sensitivity Due to Temperature, $\Delta T = T_i - T_{Ref}$ = Propellant Grain Initial Temperature – Propellant Grain Reference Temperature (70 °F)

Solid Propellant Rocket Motor Case Thickness / Weight is Usually Driven by Internal Pressure

Typical motor case is axisymmetric, with a front shortened ellipsoid dome (driving stress is at apex) and an aft cylinder body (driving stress is in hoop)

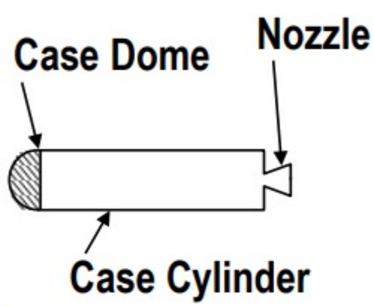


Motor Case Cylinder - Hoop Stress

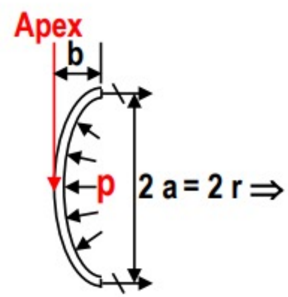


$\sigma_t \, t = - \int_0^{\pi/2} p \, r \sin\theta \, d\theta$

$(\sigma_t)_{Cylinder\ Hoop\ Stress} = p \, r / t$



Motor Dome Ellipsoid – Max Stress at Apex



Dome Apex: $(\sigma_t)_{Dome\ Longitudinal\ Stress} = (\sigma_t)_{Dome\ Hoop\ Stress} = p \, a^2 / (2 \, b \, t)$

If a = b (hemi dome of radius r) ⇒ $\sigma_{Dome\ Apex} = p \, r / (2 \, t)$

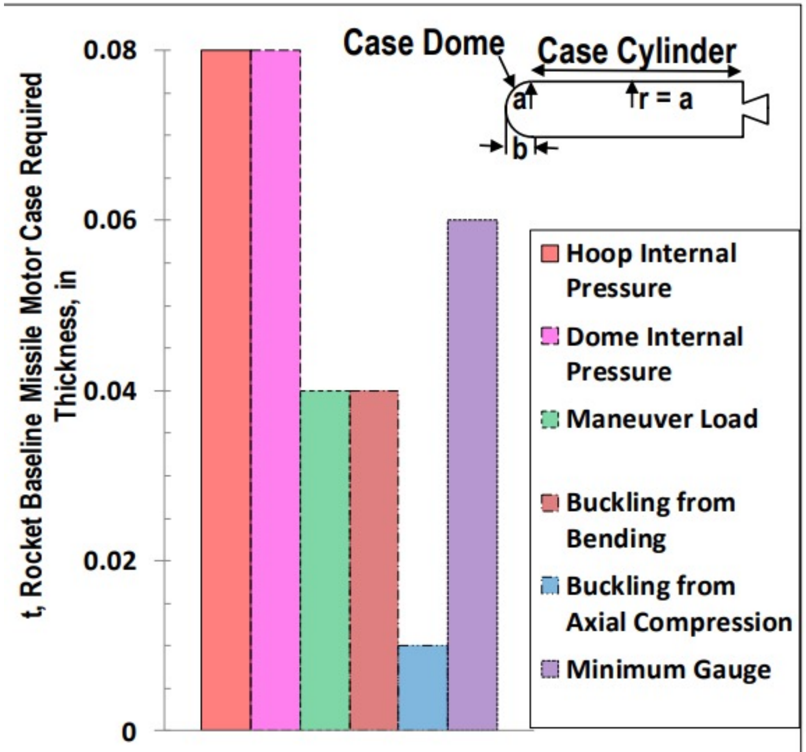
If a = 2b (shortened ellipsoid dome) ⇒ $\sigma_{Dome\ Apex} = p \, r / t = (\sigma_t)_{Cylinder\ Hoop\ Stress}$

Note:
- Equations based on thin shell: $t / r \ll 1$
- Most solid propellant rocket motors for missiles have a front shortened elliptical dome, for reduced length
- With metals – the material also reacts body bending loads (e.g., maneuver loads)
- In composite motor designs, extra (longitudinal) fibers must usually be added to accommodate body bending loads

Reference: Atkinson, J.R. and Staton, R.N., “Missile Body Weight Prediction”, SAWE 1497, May 1982
Reference: Bruhn, E.F., Orlando, J.I., and Meyers, J.F., *Analysis and Design of Missile Structures*, Tri-State Offset Company, 1967

Solid Propellant Rocket Motor Case Thickness / Weight is Usually Driven by Internal Pressure (cont)

Example: Rocket Baseline Missile Steel Motor Case



Note: Above case thickness drivers are internal pressure and maneuver loads, more important than buckling and min gauge.

Required Thickness for Rocket Baseline Missile Motor Case

- Ellipsoid Forward Dome (a / b = 2, a = r = 4 in)
- Aft Cylinder (r = Radius = 4 in)
- 4130 Steel Motor Case

$\rho = 0.283$ lb / in^3, $(\sigma_t)_{ultimate} = 190{,}000$ psi

Calculate Required Thickness from Internal Pressure

$(t_{Hoop})_{InternalPressure} = (FOS)\, p_{burst}\, r / \sigma_t = 1.5\,(2582)\,(4.0) / 190000 = 0.08$ in

$(t_{Dome\ Apex})_{InternalPressure} = (FOS)\, p_{burst}\, a^2 / (2\, b\, \sigma_t) = 1.5\,(2582)\,(4^2) / [\,2\,(2)(190000)\,] = 0.08$ in

Calculate Required Thickness from Maneuver Bending

$t_{ManeuverBending} = (FOS)\, M_B / (\pi\, \sigma\, r^2)$

Assume 30 g maneuver at launch ($W_L = 500$ lb) with uniform loading (c = 8) over length of missile (144 in) and FOS = 1.5

Then

$N = n\,W = 30\,(500) = 15{,}000$ lb

$M_B = N\, l / c = 15000\,(144) / 8 = 270{,}000$ in-lb $\Rightarrow$

$t_{ManeuverBending} = 1.5\,(270000) / [\,\pi\,(190000)\,4^2\,] = 0.04$ in

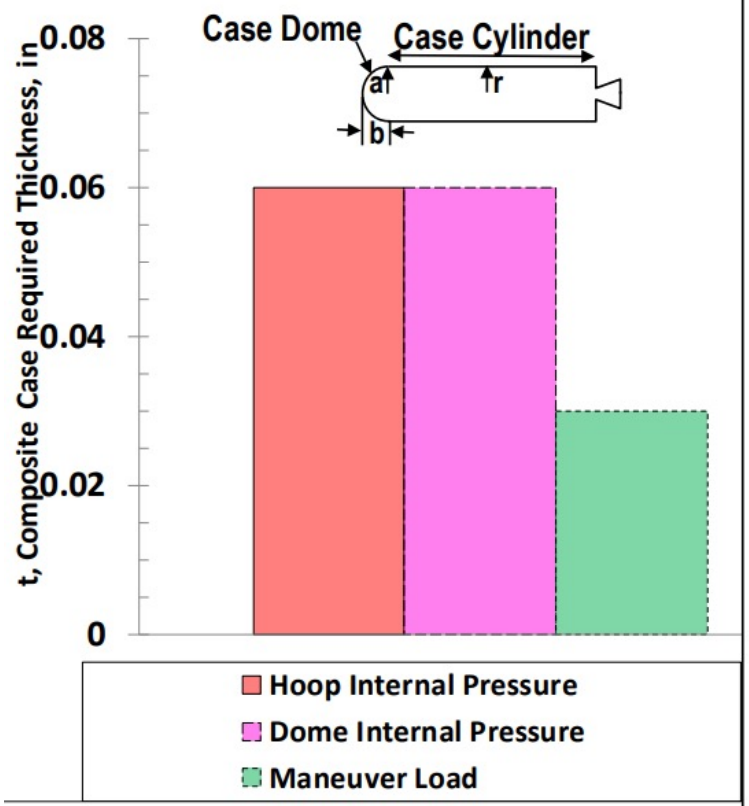
Total Required Thickness Under Combined Loads of Internal Pressure + Maneuver Load Is

$t_{Hoop} = t_{ManeuverBending} + t_{InternalPressure} = 0.04 + 0.08 = 0.12$ in

$t_{Dome} = t_{ManeuverBending} + t_{InternalPressure} = 0.04 + 0.08 = 0.12$ in

Solid Propellant Rocket Motor Case Thickness / Weight is Usually Driven by Internal Pressure (cont)

Example for Laminate Graphite Composite Case for Rocket Baseline Missile



Assume 60% Graphite / 40% Polyimide Composite Motor Case (ρ = 0. 057 lb / in^3, (σ_t)$_{ultimate}$ = 270,000 psi), r = a = 4 in, b = 2 in

Calculate thicknesses for internal pressure and maneuver load

$(t_{Hoop})_{InternalPressure} = (FOS)\ p_{burst}\ r\ /\ \sigma_t$

Calculate $(t_{Hoop})_{InternalPressure}$ = 1.5 (2582) (4.0) / 270000 = 0.06 in @ 90 deg orientation (radial fibers)

$(t_{Dome\ Apex})_{InternalPressure} = (FOS)\ p_{burst}\ a^2 / (2\ b\ \sigma_t)$ = 1.5 (2582) (4^2) / [2 (2)(270000)] = 0.06 in

Assume n = 30 g launch (W_L = 500 lb) maneuver, uniform loading (c = 8), and Factor of Safety = FOS = 1.5. Calculate

N = Normal Force = n W = 30 (500) = 15,000 lb

M_B = Bending Moment = N l / c = 15000 (144) / 8 = 270,000 in-lb $\Rightarrow$

$t_{ManeuverBending} = (FOS)\ M_B / (\pi\ \sigma\ r^2)$ = 1.5 (270000) / [π (270000) 4^2] = 0.03 in @ 0 deg orientation (longitudinal fibers)

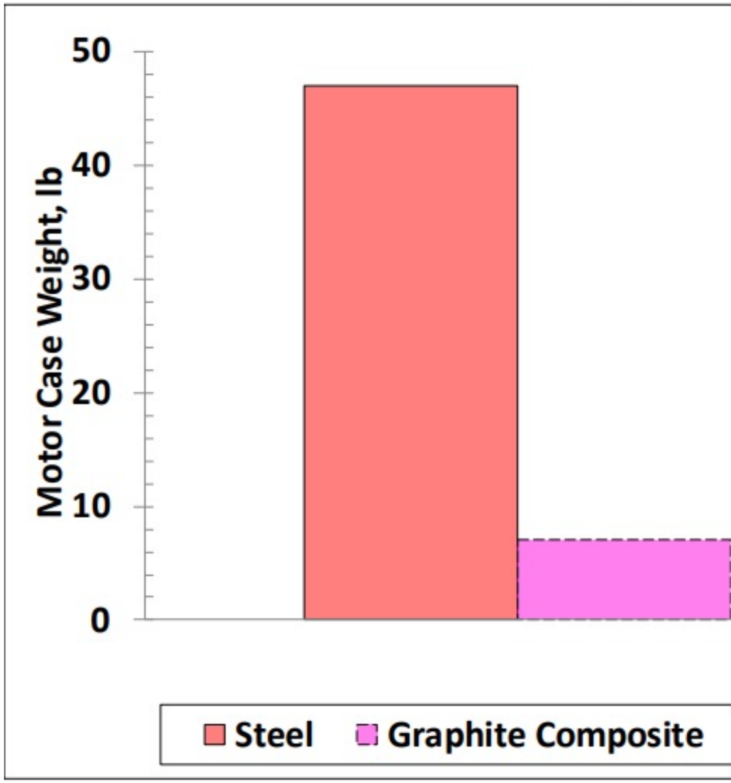
Total Required Composite Case Thickness Under Combined Loads of Internal Pressure + Maneuver Load:

$t_{Hoop} = t_{ManeuverBending} + t_{InternalPressure}$ = 0.03 @ 0 deg + 0.06 @ 90 deg = 0.09 in total

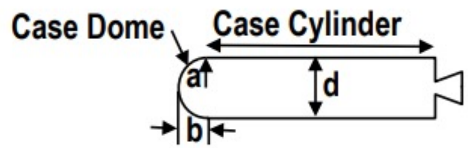
$t_{Dome} = t_{ManeuverBending} + t_{InternalPressure}$ = 0.03 longitudinal fibers + 0.06 radial fibers= 0.09 in total

Note: Composite motor case thickness is 75% of the thickness of steel baseline motor case (t = 0.09 in vs t = 0.12 in).

Laminate Graphite Composite Rocket Motor Case for Rocket Baseline Missile is Lighter Weight



Weight of Rocket Baseline Missile Motor Case



Steel Baseline Motor Case

Weight = $W_{Cylinder} + W_{Dome} = \rho\, \pi\, d\, t_{Hoop}\, l_{Cylinder} + \rho\, (2\, \pi\, a\, b)\, t_{Dome}$

= 0.283 (π) (8) (0.12) (53) + 0.283 (2) (π) (2)(4) (0.12) = 45 + 1.7 = 47 lb for steel case (w/o insulation, attachment, aft dome / nozzle)

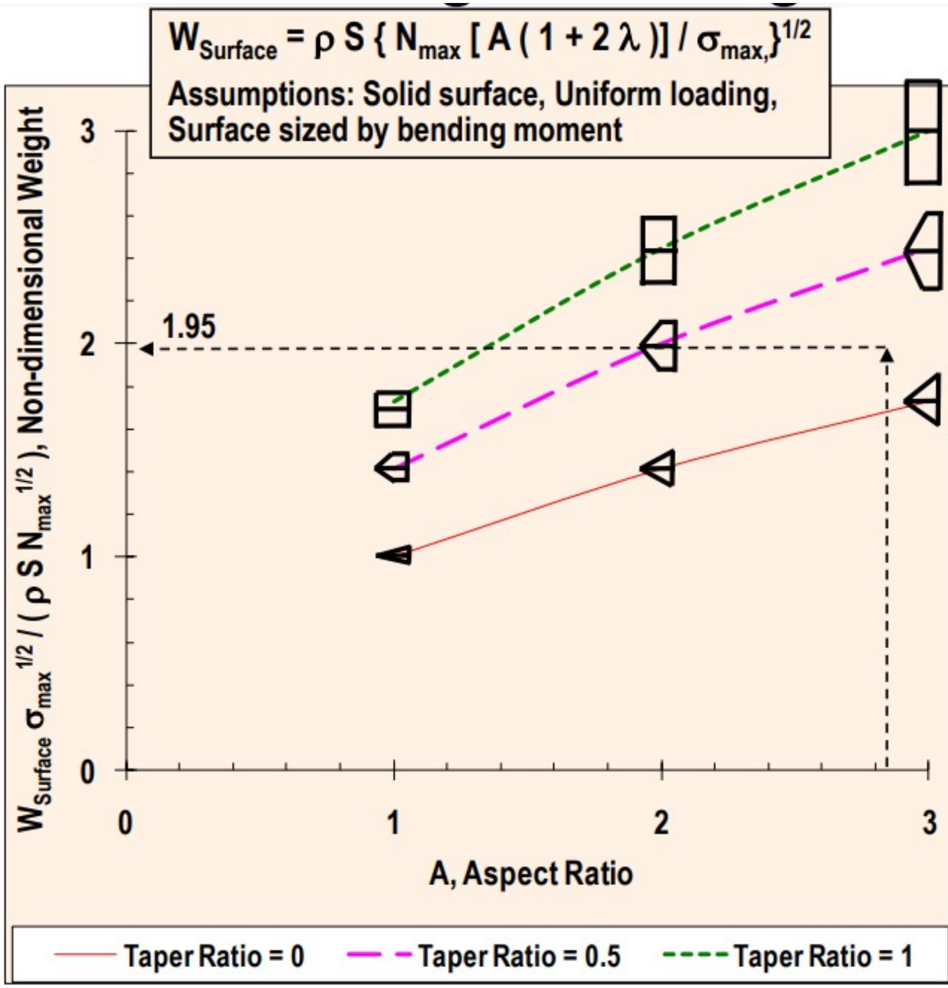
Note: From Chapter 7 data, weight = 47.3 lb

Laminate Graphite Composite Motor Case

Weight = $W_{Cylinder} + W_{Dome}$ = 0.057 (π) (8) (0.09) (53) + 0.057 (2) (π) (2)(4) (0.09) = 6.8 + 0.3 = 7.1 lb (w/o insulation, attachment, aft dome / nozzle)

Note: Composite Motor Case Weight Is 1 / 7 Weight of Steel Baseline Motor Case (7.1 lb vs 47 lb

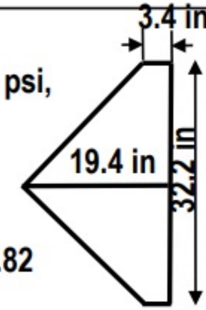
A Low Aspect Ratio Delta Surface Planform has Lighter Weight Structure



Note: Single Surface (2 panels)
$W_{Surface}$ = Surface weight sized by bending moment
ρ = Density, S = Surface area, σ_{max} = Maximum allowable (ultimate) stress, N_{max} = Maximum load, A = Aspect ratio, λ = Taper ratio

Rocket Baseline Missile Wing:

- **2219-T81 Al ($\sigma_{max} = \sigma_{ultimate}$ = 65k psi, ρ = 0.103 lbm / in^3)**
- **c_{tip} = 3.4 in, c_{root} = 19.4 in**
- **$\lambda = c_{tip} / c_{root} = 0.175$**
- **$A = b^2 / S = (32.2 \text{ in})^2 / 367 \text{ in}^2 = 2.82$**



Assume M = 2, h = 20k ft, $(\alpha + \delta)_{max}$ = 22 deg
From prior example, N_{max} = 7525 lb

Calculate

$W_{wing} \sigma_{max}^{1/2} / (\rho S N_{max}^{1/2}) = [A (1 + 2 \lambda)]^{1/2}$
$= \{ 2.82 [1 + 2 (0.175)]\}^{1/2} = 1.95$
$W_{wing} = 1.95 \rho S (N_{max} / \sigma_{max})^{/2}$
$= 1.95 (0.103)(367)(7525 / 65000)^{1/2}$
= 25.1 lb for 1 wing

Note: Rocket baseline missile wings are cruciform (2 wings). Weight of 2 wings = 2 x 25.1 = 50.2 lb

Multi-mode Seeker Dome Material is Driven by RF / IR Transmission and Flight Enviroment

Multi-Mode Seeker Dome Material	Density (g / cm^3)	Dielectric Constant	SWIR / MWIR and LWIR Bandpass		Transverse Strength (10^3 psi)	Thermal Expansion (10^{-6} / °F)	Erosion, Knoop (kg / mm^2)	Max Temp (°F) w/o EO / EM Degradation
Zinc Sulfide (Z_nS)	4.05	8.4 Average	Superior	Superior	18 Average	4 Average	350 Average	700 Average
Sapphire (Al_2O_3) / Spinel ($MgAl_2O_4$)	3.68	8.5 Average	Superior	Poor	28 Average	3 Average	1650 Superior	1800 Superior
Quartz / Fused Silica (S_iO_2)	2.20	3.7 Superior	Average	Very Poor	8 Poor	0.3 Superior	600 Good	2000 Superior
Diamond (C)	3.52	5.6 Good	Average	Superior	400 Superior	1 Superior	8800 Superior	3500 Superior
Mag. Fluoride (M_gF_2)	3.18	5.5 Good	Superior	Poor	7 Poor	6 Poor	420 Good	1000 Good

● Superior ◒ Good ○ Average ◡ Poor ▪ Very Poor

Note:
RF = Radar Frequency, IR = Infrared, SWIR = Short Wave Infrared, MWIR = Mid Wave Infrared, LWIR = Long Wave Infrared, EO = Electro Optical, EM = Electro Magnetic

Infrared Seeker Dome Material is Driven by IR Transmission and Flight Environment

Infrared Seeker Dome Material	Density (g / cm^3)	SWIR / MWIR and LWIR Bandpass		Transverse Strength (10^3 psi)		Thermal Expansion (10^{-6} / °F)		Erosion, Knoop (kg / mm^2)		Max Temp (°F) w/o EO Degradation	
Zinc Sulfide (Z_nS)	4.05	Superior	Superior	18	Average	4	Average	350	Average	700	Average
Zinc Selenide (Z_nS_e)	5.16	Superior	Superior	8	Poor	4	Average	150	Average	600	Average
Mag. Fluoride (M_gF_2)	3.18	Superior	Poor	7	Poor	6	Poor	420	Good	1000	Good
Germanium (Ge)	5.33	Average	Average	15	Average	4	Average	780	Good	200	Poor
Sapphire (Al_2O_2) / Spinel ($MgAl_2O_4$)	3.68	Superior	Poor	28	Average	3	Average	1650	Superior	1800	Superior
Diamond (C)	3.52	Average	Superior	400	Superior	1	Superior	8800	Superior	3500	Superior
Alon ($Al_{23}O_{27}N_5$)	3.67	Superior	Very Poor	44	Good	3	Average	1900	Superior	1800	Superior
Quartz / Fused Silica (S_iO_2)	2.20	Average	Very Poor	8	Poor	0.3	Superior	600	Good	2000	Superior
Yttria (Y_2O_3)	5.01	Superior	Poor	23	Average	4	Average	700	Good	1800	Superior

● Superior ◒ Good ○ Average ◡ Poor ▪ Very Poor

Radar Seeker Radome Material is Driven by RF Transmission and Flight Enviornment

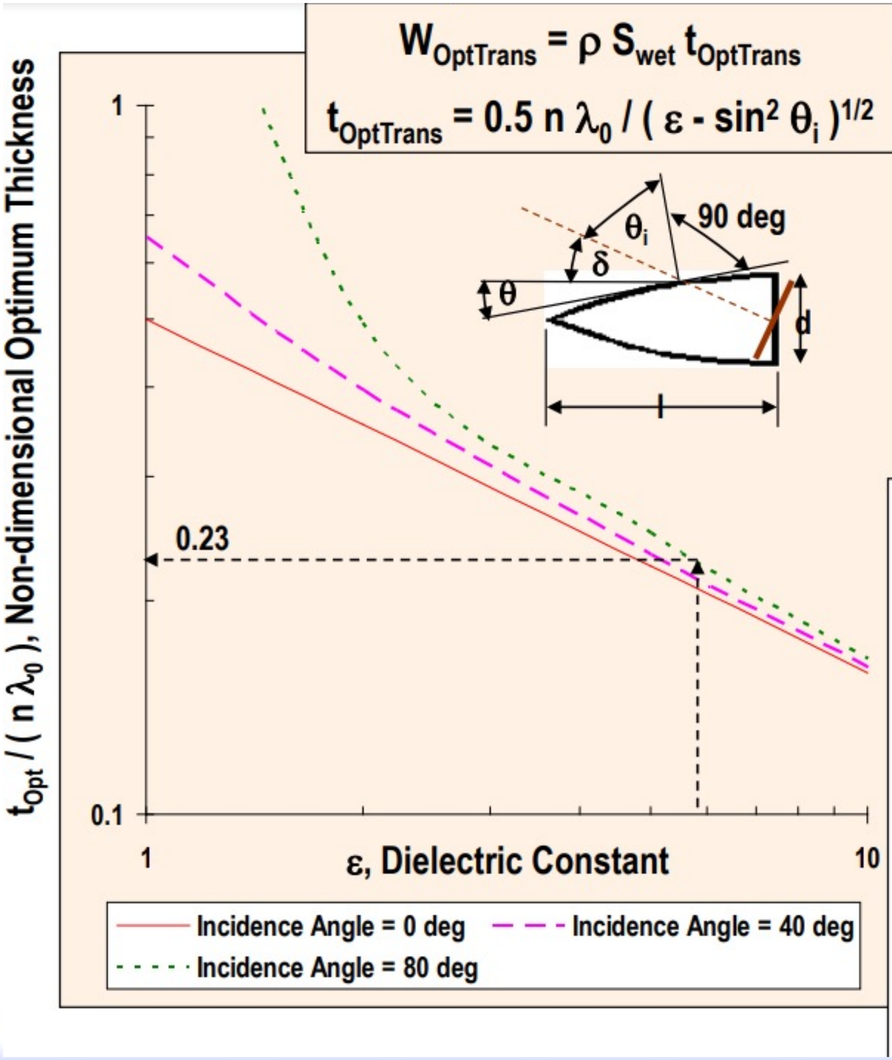
Radar Seeker Dome Material	Density (g / cm^3)	Dielectric Constant	Transverse Strength (10^3 psi)	Thermal Expansion (10^{-6} / °F)	Erosion, Knoop (kg / mm^2)	Max Temp (°F) w/o EM Degradation
Quartz / Fused Silica (SiO_2)	2.20	3.7 ●	8 ◡	0.3 ●	600 ◒	2000 ●
Silicon Nitride (Si_3N_4)	3.18	6.1 ◒	90 ●	2 ◒	2200 ●	2700 ●
Diamond (C)	3.52	5.6 ◒	400 ●	1 ●	8800 ●	3500 ●
Pyroceram	2.55	5.8 ◒	25 ○	3 ○	700 ◒	2200 ●
Polyimide	1.54	3.2 ●	17 ○	40 ◡	70 ◡	700 ○
Mag. Fluoride (MgF_2)	3.18	5.5 ◒	7 ◡	6 ◡	420 ◒	1000 ◒

● Superior ◒ Good ○ Average ◡ Poor ▪ Very Poor

A Driver for Radome Weight is the Optimum Thickness Required for Efficient Transmission

$$W_{OptTrans} = \rho\, S_{wet}\, t_{OptTrans}$$

$$t_{OptTrans} = 0.5\, n\, \lambda_0 / (\varepsilon - \sin^2 \theta_i)^{1/2}$$



Nomenclature:

$W_{OptTrans}$ = Weight for Optimum Transmission
ρ = Density
S_{wet} = Surface wetted area
$t_{OptTrans}$ = Optimum thickness for 100% transmission
n = Integer (0, 1, 2, …)
λ_0 = Wavelength in air
ε = Dielectric constant
θ_i = Radar signal incidence angle = 90 deg - δ - θ
θ = Surface local angle
δ = Seeker look angle

Example: Rocket Baseline Missile Pyroceram Radome

$\varepsilon = 5.8$, $\rho = 0.092$ lbm / in^3, $\lambda_0 = 1.1$ in, tangent ogive, l = 19.2 in, d = 8 in, $S_{wet} = 326$ in^2

$\delta = 0$ deg $\Rightarrow (\theta_i)_{avg} \approx 90 - 0 - 11.8 = 78.2$ deg

$t_{OptTrans} / (n\, \lambda_0) = 0.5 / (5.8 - 0.96)^{1/2} = 0.23$

Assume n = 1 (optimum transmission)

$t_{OptTrans} = 0.23\, n\, \lambda_0 = 0.23 (1)(1.1) = 0.25$ in
$W_{OptTrans} = 0.092 (326) (0.25) = 7.5$ lb (3.4 kg)

Assume n = 0.3 (lighter weight)

$t_{OptTrans} = 0.23\, n\, \lambda_0 = 0.23 (0.3) (1.1) = 0.075$ in
$W_{OptTrans} = 0.092 (326) (0.075) = 2.2$ lb (1.0 kg)

Missile Electrical Power Supply Drivers Include Weight, Environment, and Safety

Measure of Merit	Generator	Lithium Battery (Non-Rechargeable)	Lithium Battery (Rechargeable)	Thermal Battery
Weight for Long Time of Flight	Superior	Average	Average	Average
Weight for Short Time of Flight	Average	Average	Average	Superior
Max Acceleration	Poor	Good	Good	Superior
Storage Life	Superior	Average / Good	Poor / Good	Superior
Voltage Stability	Average	Superior	Superior	Average
Max / Min Temp	Average	Poor	Poor	Average
Safety	Superior	Average	Average	Good

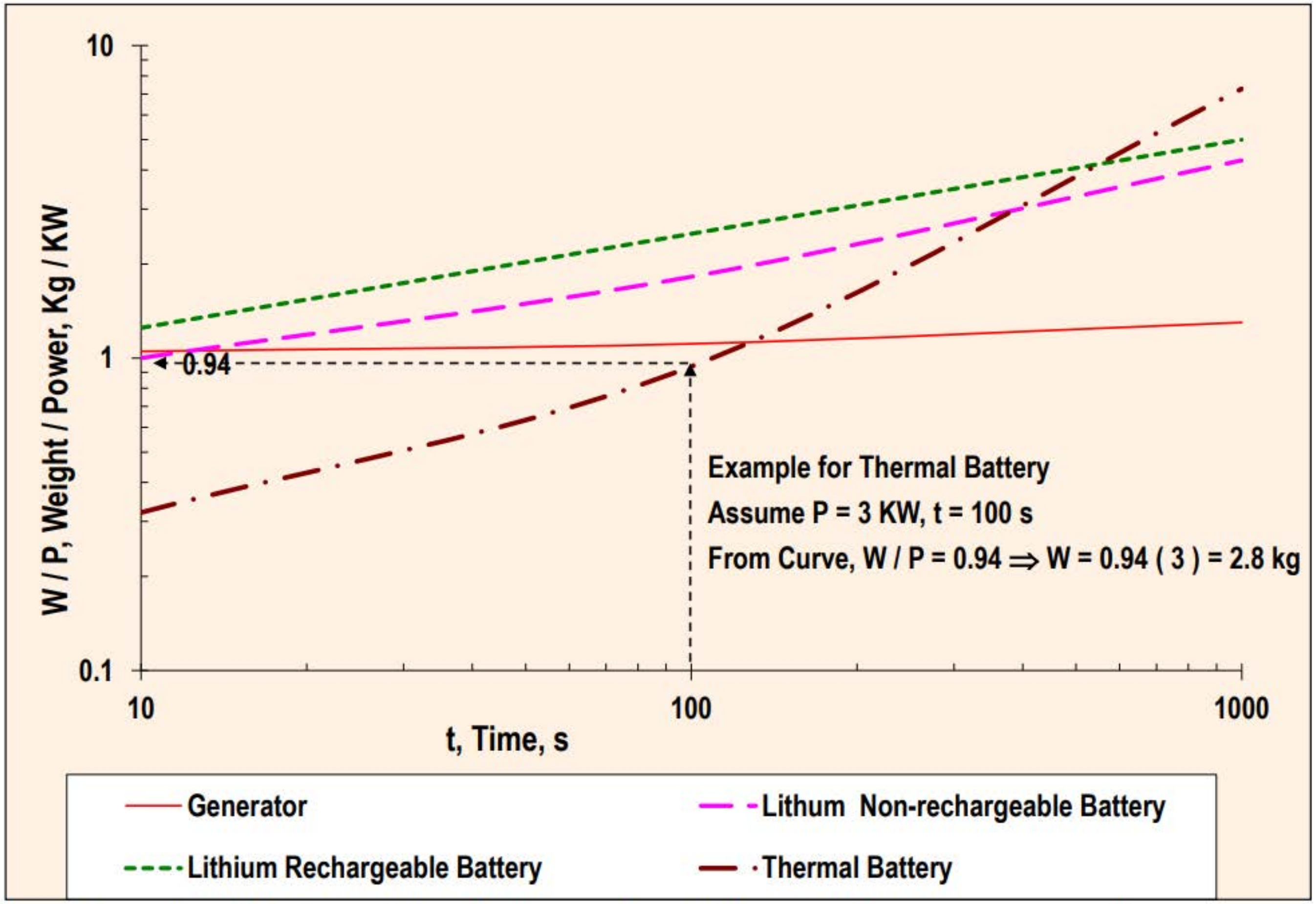
Superior Good Average Poor

Note: Generator provides highest energy with light weight for long time of flight (e.g., cruise missile)

Lithium battery provides nearly constant voltage suitable for electronics. Relatively high energy with light weight

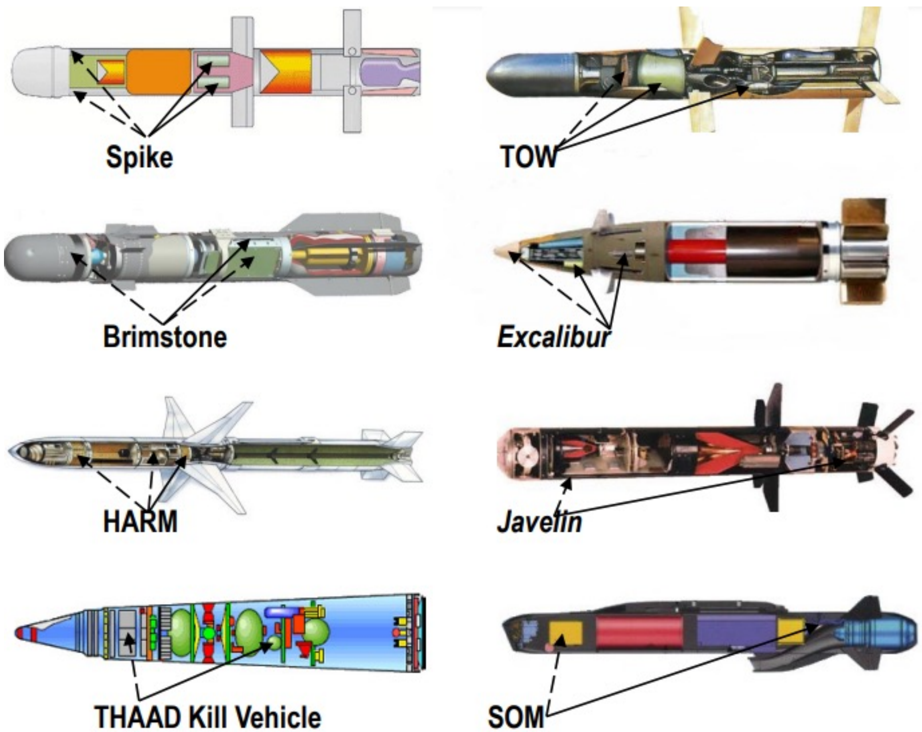
Thermal battery provides highest power with light weight for short time of flight (most popular type of battery for missiles)

A Thermal Battery has Lighter Weight for Short Time of Flight – Generator has Lighter Weight for Long TOF



Note: Missiles typically have short time of flight ⇒ battery weight usually driven more by power requirement than by energy

Missile Power Supply is Usually a Thermal Battery and Most Batteries and Located Near Electronics



Note: Above missiles have thermal batteries.

⟶ Thermal Battery

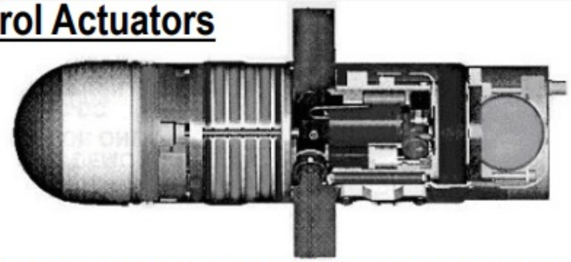
– – ➝ Electronics

TUBITAK-SAGE Video: Thermal Battery for SOM

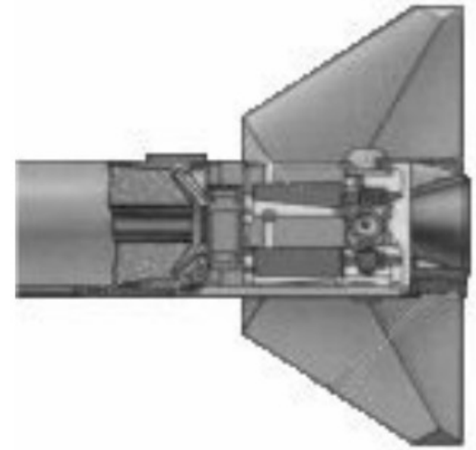
Most Missiles use Electromechanical Flight Control Actuators

Examples of EM Flight Control Actuators

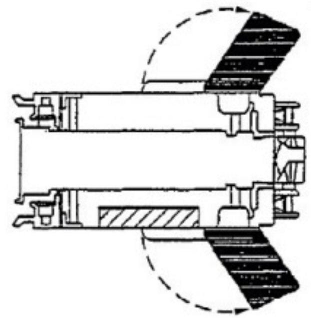
Canard (Stinger) ....



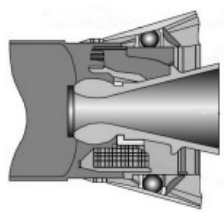
Tail (AMRAAM) ....



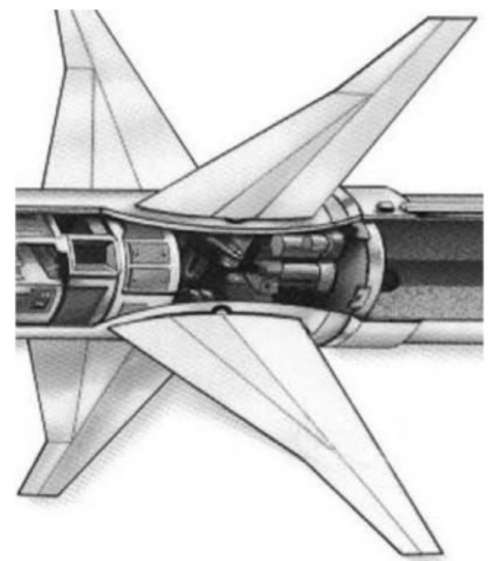
Jet Vane / Tail (Javelin) ....



Movable Nozzle (THAAD) ...



Wing (HARM) ....



Note: Electromechanical (EM) flight control actuators are more reliable than the actuator alternatives of hydraulic, cold gas pneumatic, or warm gas pneumatic EM actuators typically have high torque, light weight, high deflection rate, high bandwidth, and efficient packaging