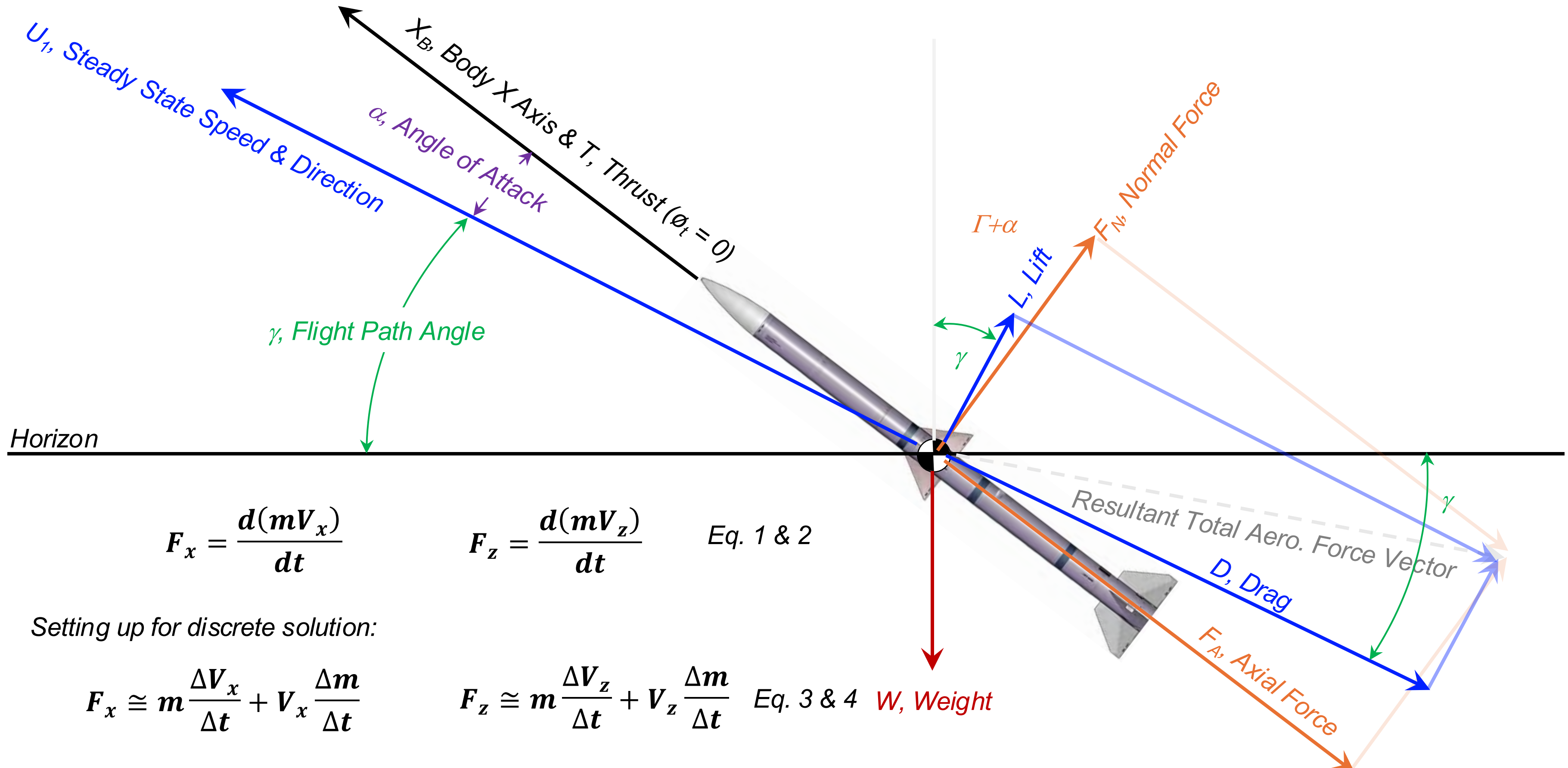
1st Order Proverse Engineering



$$F_x = \frac{d(mV_x)}{dt} \qquad F_z = \frac{d(mV_z)}{dt}$$ Eq. 1 & 2

Setting up for discrete solution:

$$F_x \cong m\frac{\Delta V_x}{\Delta t} + V_x\frac{\Delta m}{\Delta t} \qquad F_z \cong m\frac{\Delta V_z}{\Delta t} + V_z\frac{\Delta m}{\Delta t}$$ Eq. 3 & 4

$$F_x = TCos(\alpha + \phi_t + \gamma) - C_D qACos\gamma - C_L qASin\gamma \cong m\frac{\Delta V_x}{\Delta t} + V_x\frac{\Delta m}{\Delta t}$$ Eq. 5

$$F_z = TSin(\alpha + \phi_t + \gamma) + C_L qACos\gamma - C_D qASin\gamma - W \cong m\frac{\Delta V_z}{\Delta t} + V_z\frac{\Delta m}{\Delta t}$$ Eq. 6

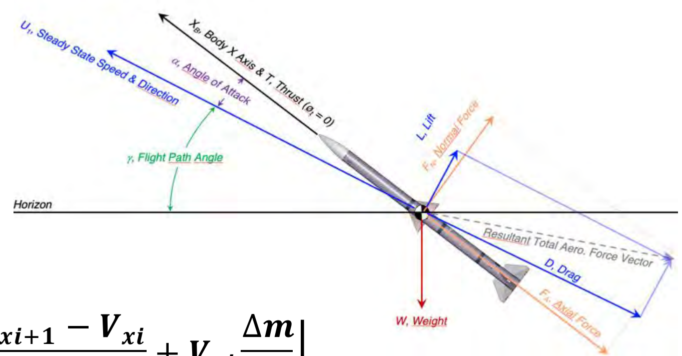
1st Order Proverse Engineering

Step 1: Get Launch Conditions: x_o, z_o coordinates, V_{xo}, V_{zo}, m_o, atmospherics

Note the velocities are often nonzero considering aircraft launch or tube launch from ground

Step 2: Find or determine information on powerplant: $T(t)$, $I_{sp}(t)$, $m_{propellant}$

Step 3: Assume (initially) an angle of attack, α, and thrust angle, $\varnothing_t$



Step 4: Solve for initial flight path angle, γ (from initial information)

Step 5: Break down equations 5 and 6 for discretization:

$$F_{xi} = T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \frac{V_{xi+1} - V_{xi}}{\Delta t} + V_{xi} \frac{\Delta m}{\Delta t}\Big|_i$$

$$F_{zi} = T_i Sin(\alpha_i + \phi_{ti} + \gamma_i) + C_{Li} q_i A Cos\gamma_i - C_{Di} q_i A Sin\gamma_i - W_i \cong m_i \frac{V_{zi+1} - V_{zi}}{\Delta t} + V_{zi} \frac{\Delta m}{\Delta t}\Big|_i$$

Step 6: Solve for x_{i+1}, z_{i+1} m_{i+1}, V_{xi+1}, V_{zi+1}, V_{total} (U_1), q_{i+1}(using V_{total}), C_{Li+1}, C_{Di+1}, γ_{i+1}

Step 7: Adjust α_{i+1}, $\varnothing_{i+1}$ and T_{i+1} as needed

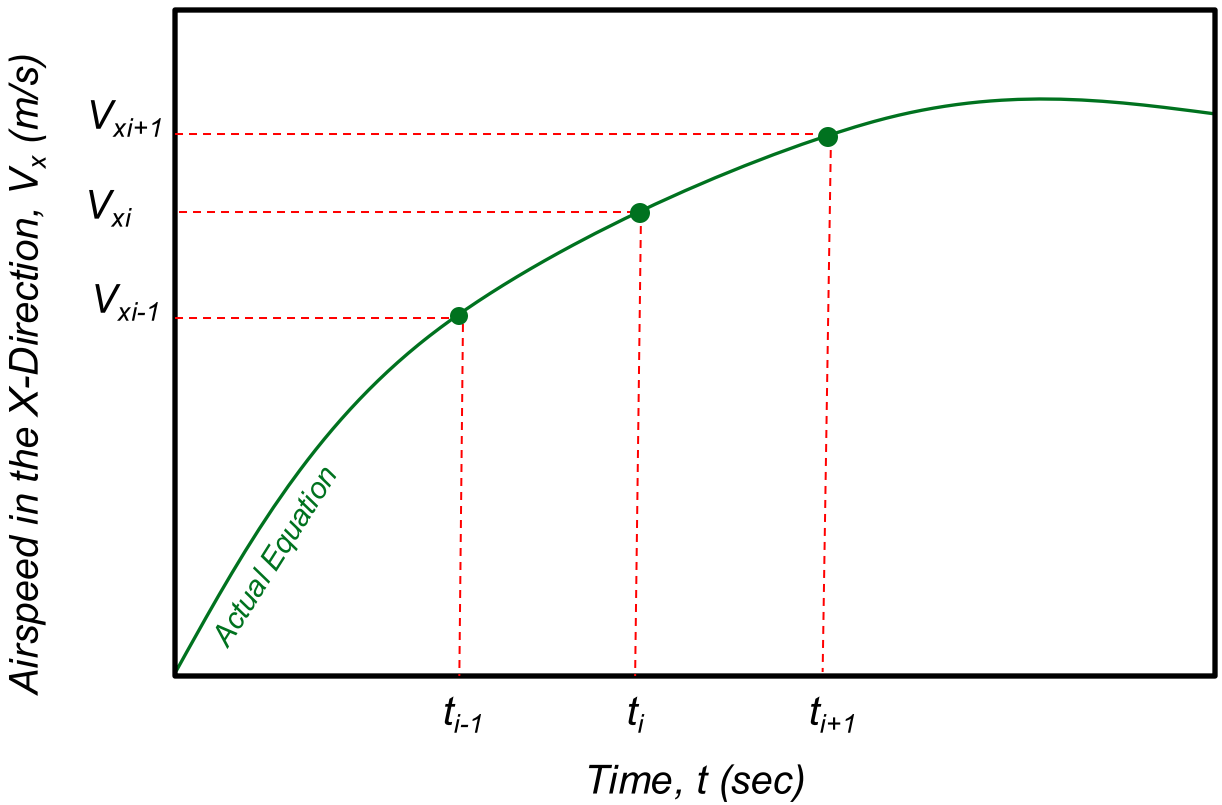
Step 8: Repeat

1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

1st Order Difference Expression



1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

Recall the Taylor Series Expansion:

$$f(x_{i+1}) = f(x_i) + \Delta x\, f'(x_i) + \frac{\Delta x^2}{2!} f''(x_i) + \frac{\Delta x^3}{3!} f'''(x_i) + \ldots + \frac{\Delta x^n}{n!} f^n(x_i) + R_n$$

$$R_n = \frac{\Delta x^{n+1}}{n+1!} f^{(n+1)}(\xi)$$

.

1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

Recall the Taylor Series Expansion

$$f(x_{i+1}) = f(x_i) + \Delta x\, f'(x_i) + \frac{\Delta x^2}{2!} f''(x_i) + \frac{\Delta x^3}{3!} f'''(x_i) + \ldots + \frac{\Delta x^n}{n!} f^n(x_i) + R_n$$

$$R_n = \frac{\Delta x^{n+1}}{n+1!} f^{(n+1)}(\xi)$$

Use this to get an estimate for $f'(x_i)$: $\left. f(x_{i+1}) = f(x_i) + \Delta x\, f'(x_i) + \frac{\Delta x^2}{2!} f''(x_i) + O(\Delta x^3) \right\}$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} - \frac{\Delta x}{2!} f''(x_i) + O(\Delta x^2)$$

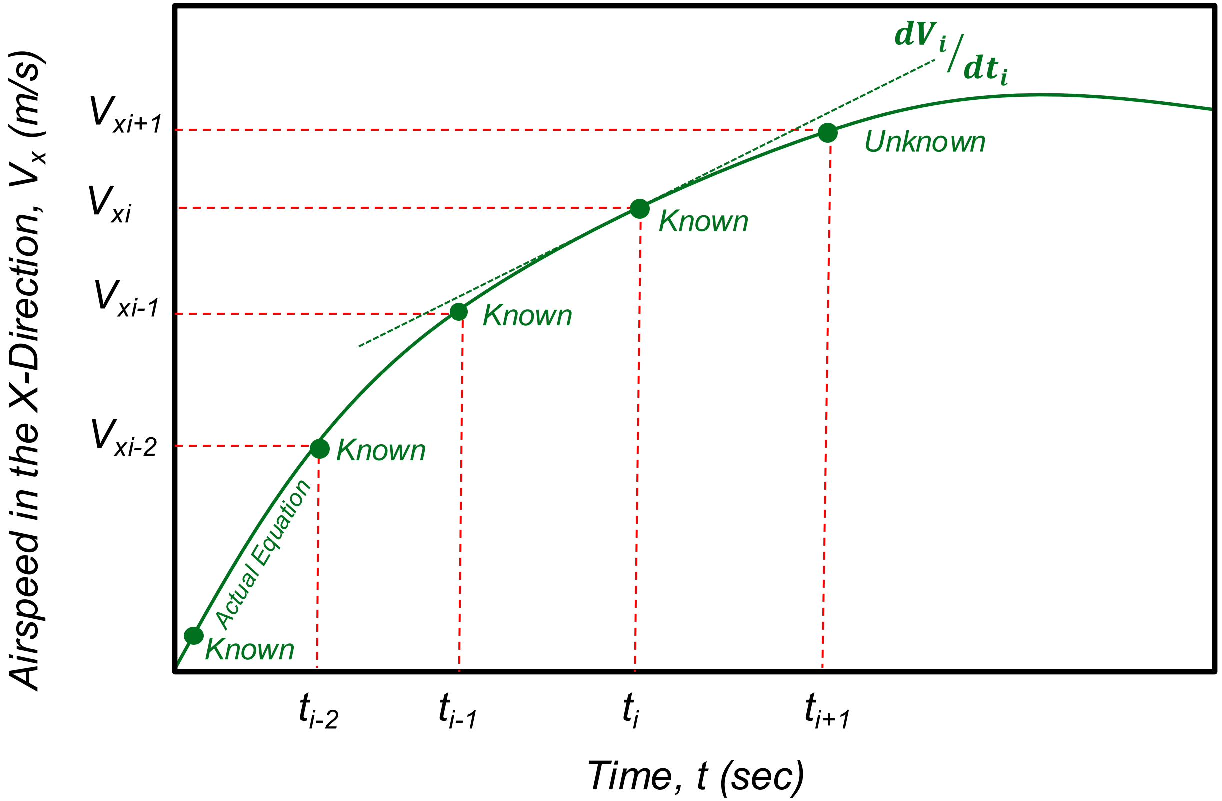
.

1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

1st Order Difference Expression $\quad \frac{dV}{dt} \cong \frac{\Delta V}{\Delta t}$

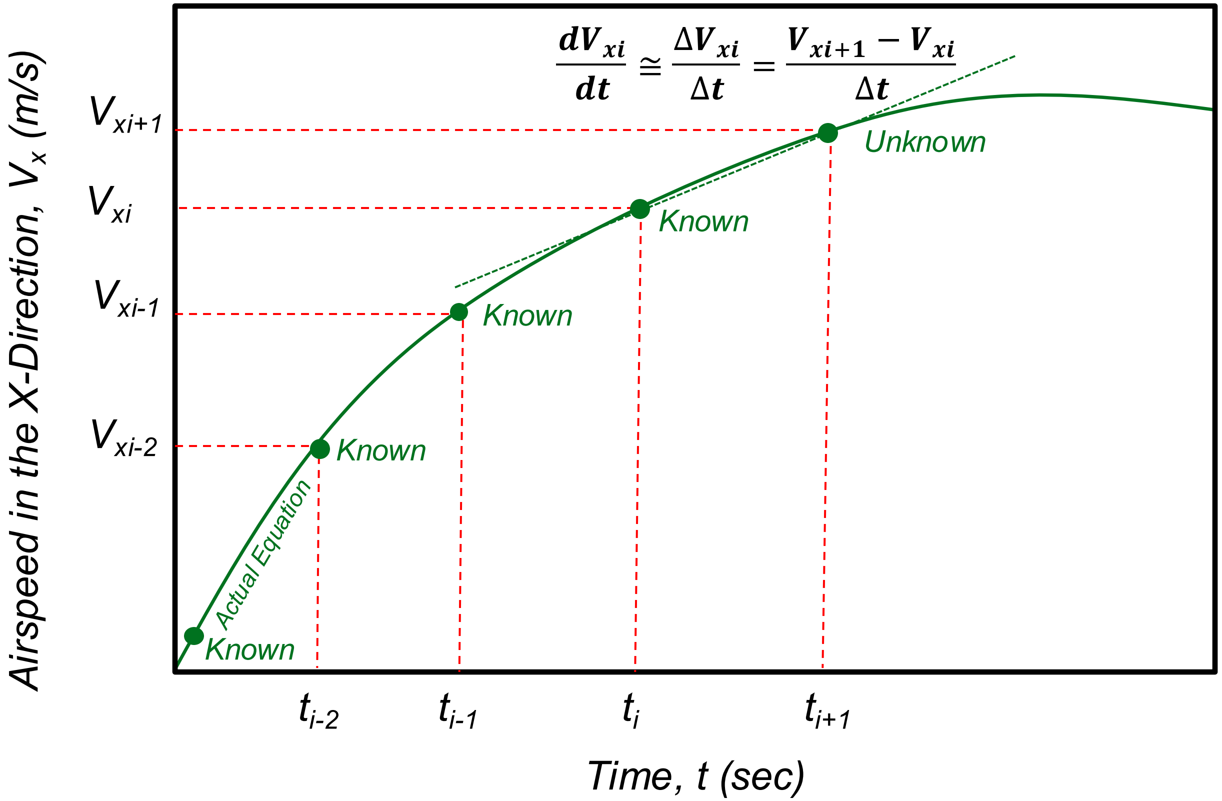


1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

1st Order Forward Difference

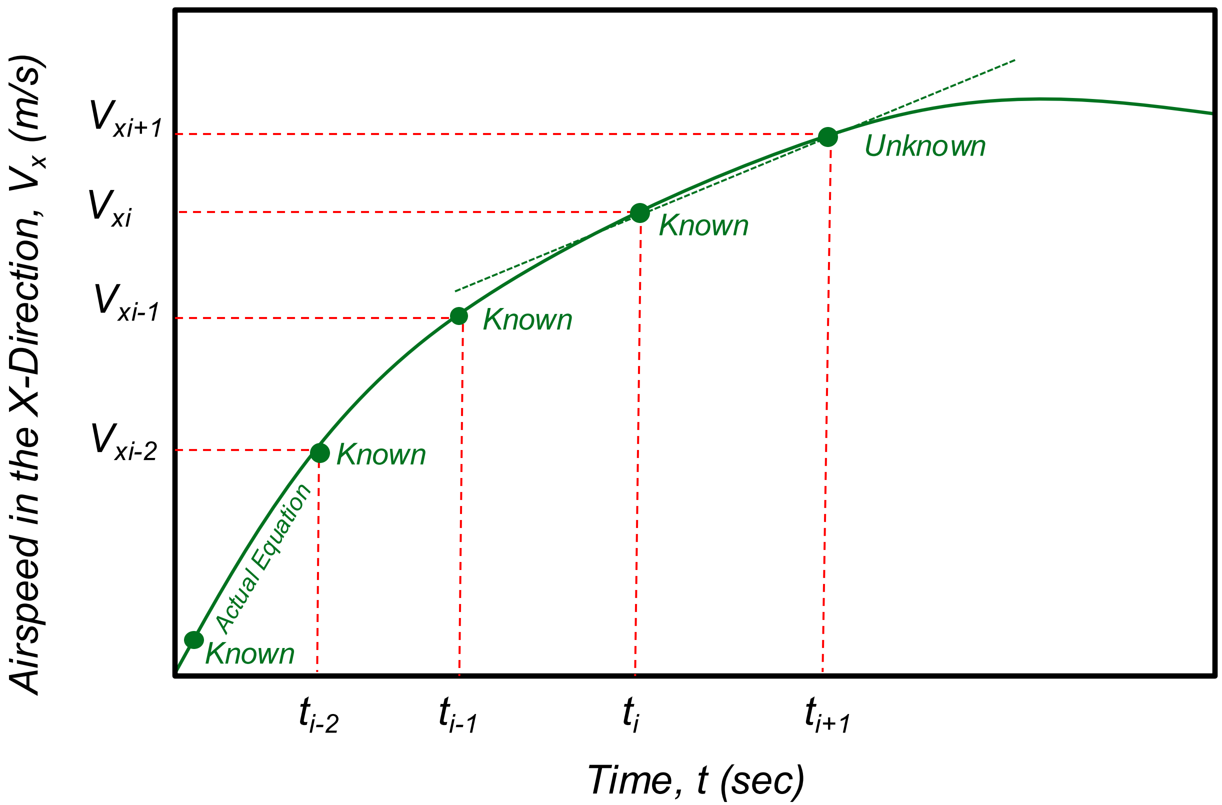


1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

1st Order Forward Difference

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{V_{xi+1} - V_{xi}}{\Delta t}$$

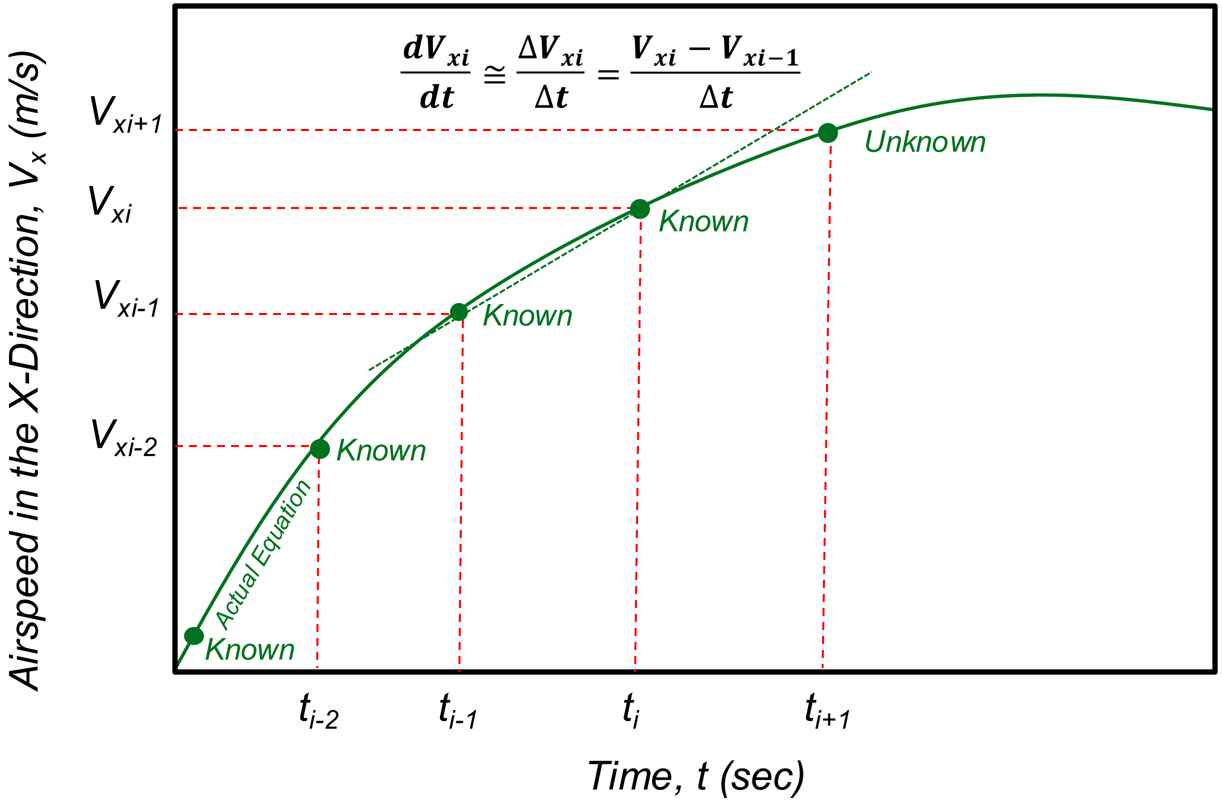


1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

1st Order Backwards Difference

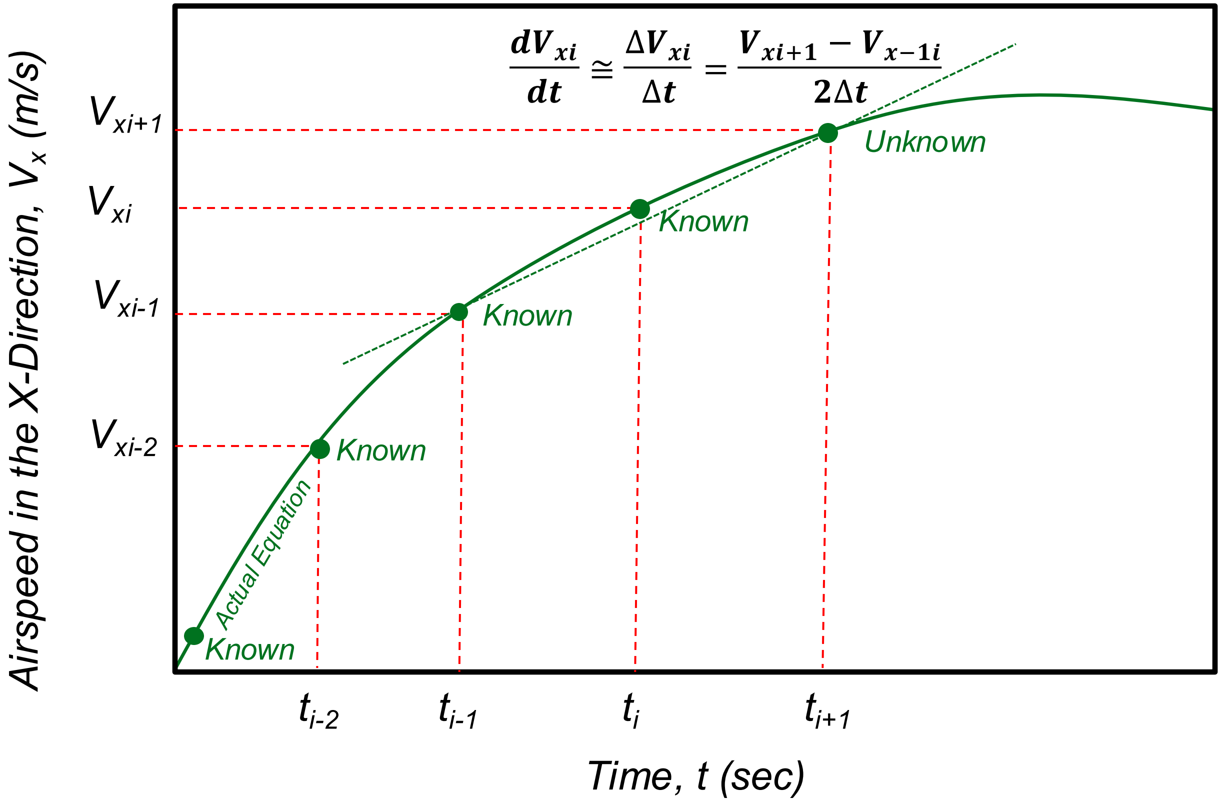


1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

1st Order Centered Difference



1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

Higher Order Accuracy:

$$f(x_{i+1}) = f(x_i) + \Delta x\, f'(x_i) + \frac{\Delta x^2}{2!} f''(x_i) + O(\Delta x^3) \quad *(-2)$$

$$f(x_{i+2}) = f(x_i) + 2\Delta x\, f'(x_i) + \frac{4\Delta x^2}{2!} f''(x_i) + O(\Delta x^3) \quad *(1)$$

$$\Rightarrow f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{\Delta x^2} + O(\Delta x)$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} - \frac{\Delta x}{2!} f''(x_i) + O(\Delta x^2)$$

$$\Rightarrow f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} - \frac{\Delta x}{2!}\frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{\Delta x^2} + O(\Delta x^2) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2\Delta x} + O(\Delta x^2)$$

.

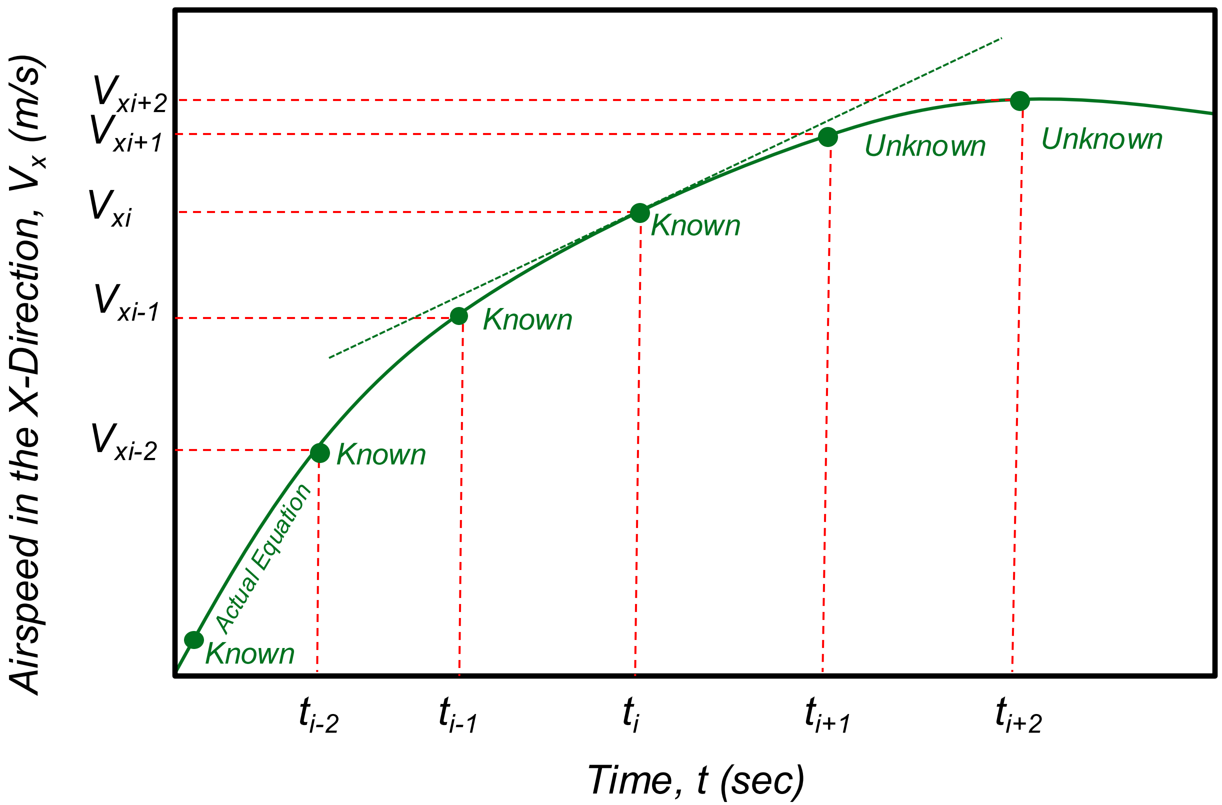
1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

2nd Order Forward Difference

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{-V_{xi+2} + 4V_{xi+1} - 3V_{xi}}{2\Delta t}$$



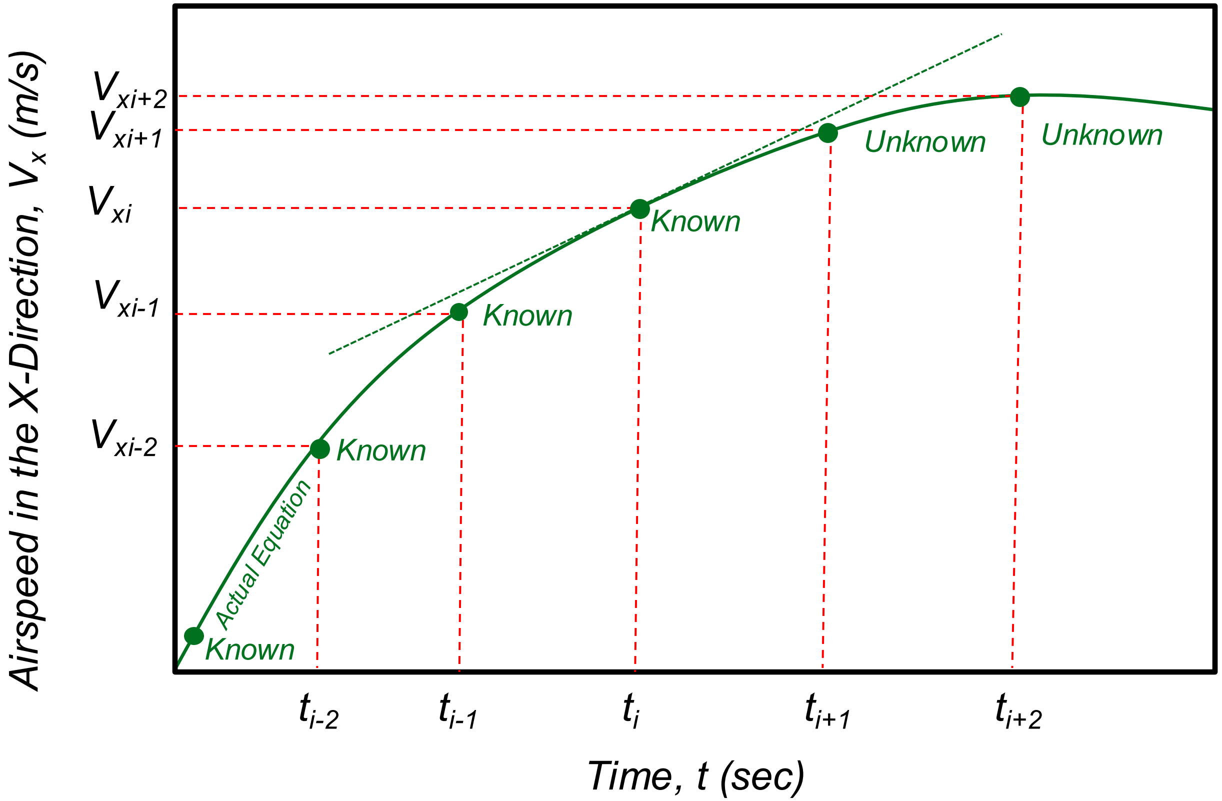
1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

Decremented
2nd Order Forward Difference

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{-V_{xi+1} + 4V_{xi} - 3V_{xi-1}}{2\Delta t}$$



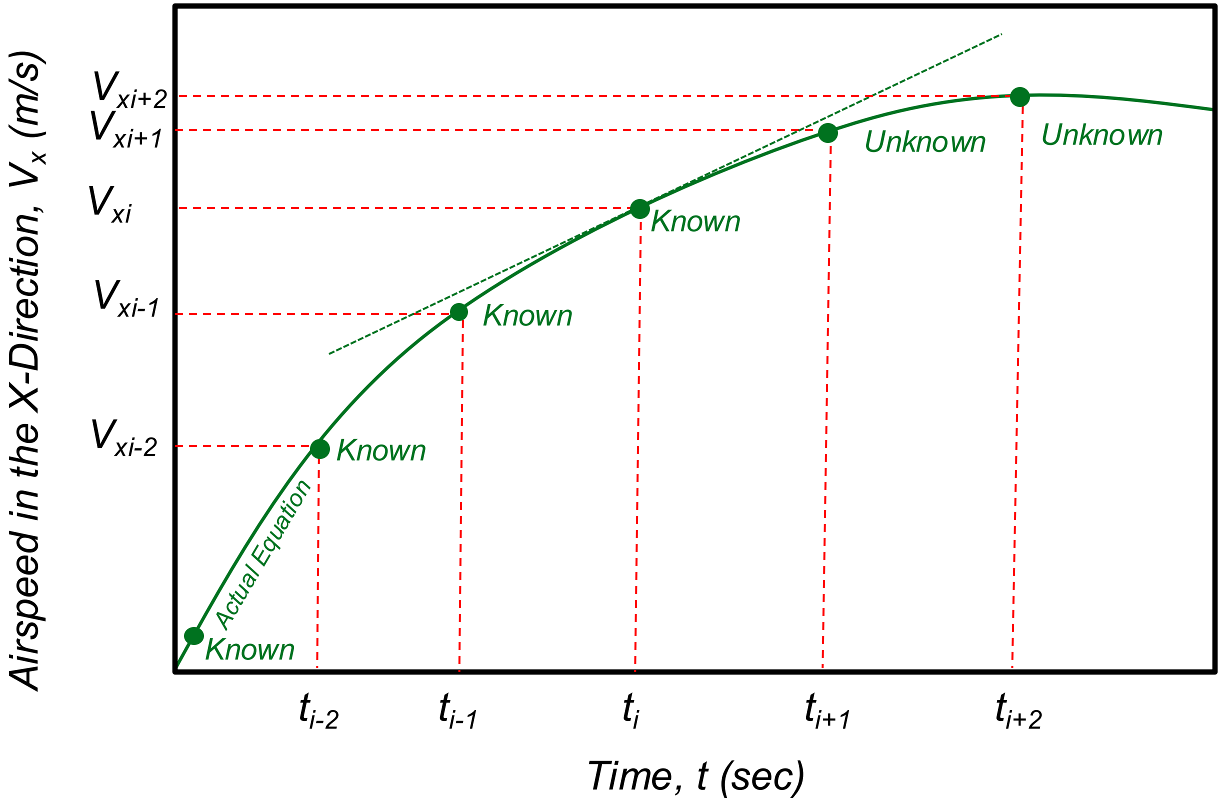
1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

Padé Approximation
4th Order Center Difference

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{-V_{xi+2} + 8V_{xi+1} - 8V_{xi-1} + V_{xi-2}}{12\Delta t}$$



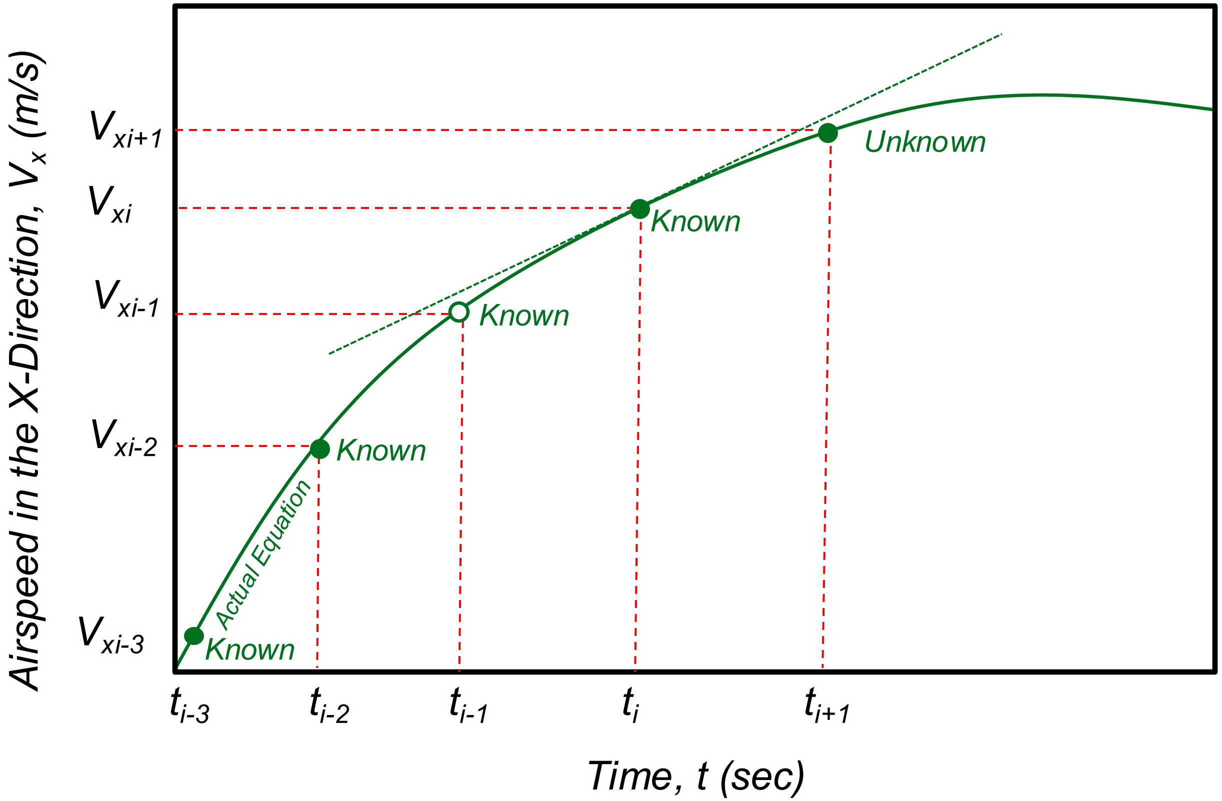
1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$F_{xi} \cong T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left.\frac{\Delta V}{\Delta t}\right|_i + V_{xi} \left.\frac{\Delta m}{\Delta t}\right|_i$$

Padé Approximation Decremented 4th Order Forward Difference

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{-V_{xi+1} + 8V_{xi} - 8V_{xi-2} + V_{xi-3}}{12\Delta t}$$



1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left(\frac{-V_{xi+1} + 8V_{xi} - 8V_{xi-2} + V_{xi-3}}{12\Delta t} \right) + V_{xi} \frac{\Delta m}{\Delta t} \bigg|_i$$

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Recall: I_{sp}

$$I_{sp} = \frac{T}{\dot{W}_f} = \frac{T}{g \dot{m}_f} = \frac{T}{g \, {dm_f}/{dt}} \cong \frac{T}{g \, {\Delta m_f}/{\Delta t}} \quad \therefore \quad \frac{\Delta m}{\Delta t}\bigg|_i = \frac{T}{\dot{W}_f} = \frac{T}{g \dot{m}_f} = \frac{T}{g \, {dm_f}/{dt}} \cong \frac{T_i}{g I_{spi}}$$

1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

$$T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left(\frac{-V_{xi+1} + 8V_{xi} - 8V_{xi-2} + V_{xi-3}}{12\Delta t} \right) + V_{xi} \frac{\Delta m}{\Delta t}\bigg|_i$$

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$$T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left(\frac{-V_{xi+1} + 8V_{xi} - 8V_{xi-2} + V_{xi-3}}{12\Delta t} \right) + V_{xi} \frac{T_i}{g I_{spi}}$$

1st Order Proverse Engineering

Primer on Finite Difference Techniques & Numerical Solution

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$$T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left(\frac{-V_{xi+1} + 8V_{xi} - 8V_{xi-2} + V_{xi-3}}{12\Delta t} \right) + V_{xi} \frac{\Delta m}{\Delta t} \bigg|_i$$

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$$T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) - C_{Di} q_i A Cos\gamma_i - C_L q A Sin\gamma_i \cong m_i \left(\frac{-V_{xi+1} + 8V_{xi} - 8V_{xi-2} + V_{xi-3}}{12\Delta t} \right) + V_{xi} \frac{T_i}{g I_{spi}}$$

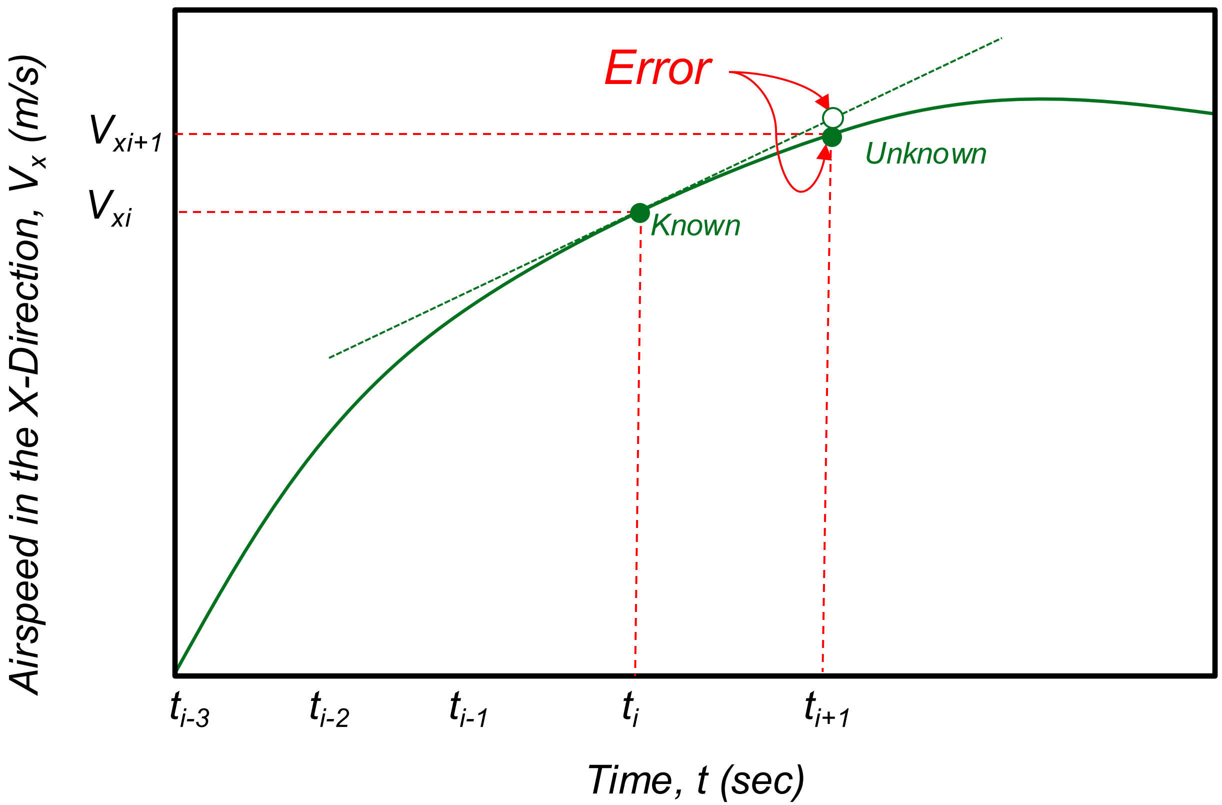
$$V_{xi+1} = \frac{12\Delta t}{m_i} \left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi} \frac{T_i}{g I_{spi}} \right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

1st Order Proverse Engineering

1st Order approximations for V_{i+1}

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{V_{xi+1} - V_{xi}}{\Delta t} \qquad \therefore \qquad V_{xi+1} = \left.\frac{dV_x}{dt}\right|_i \Delta t + V_{xi}$$

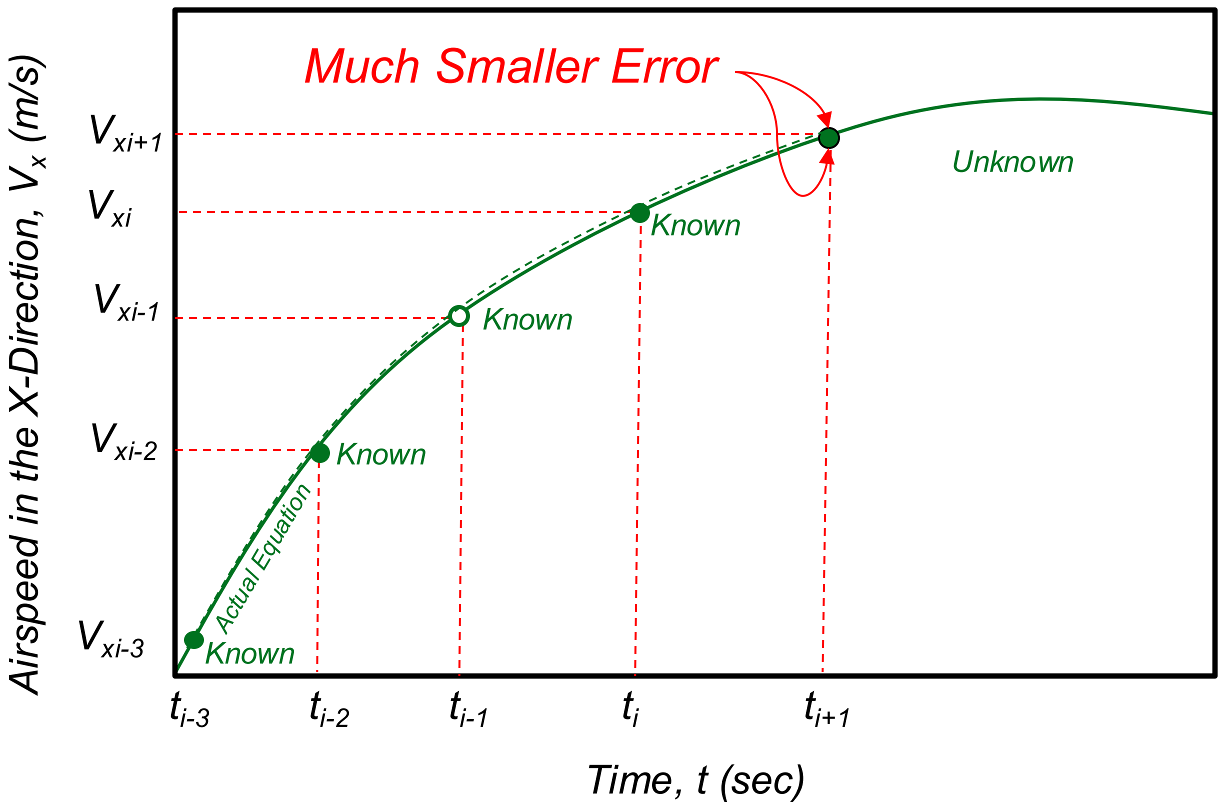
Error between estimate and exact expression



1st Order Proverse Engineering

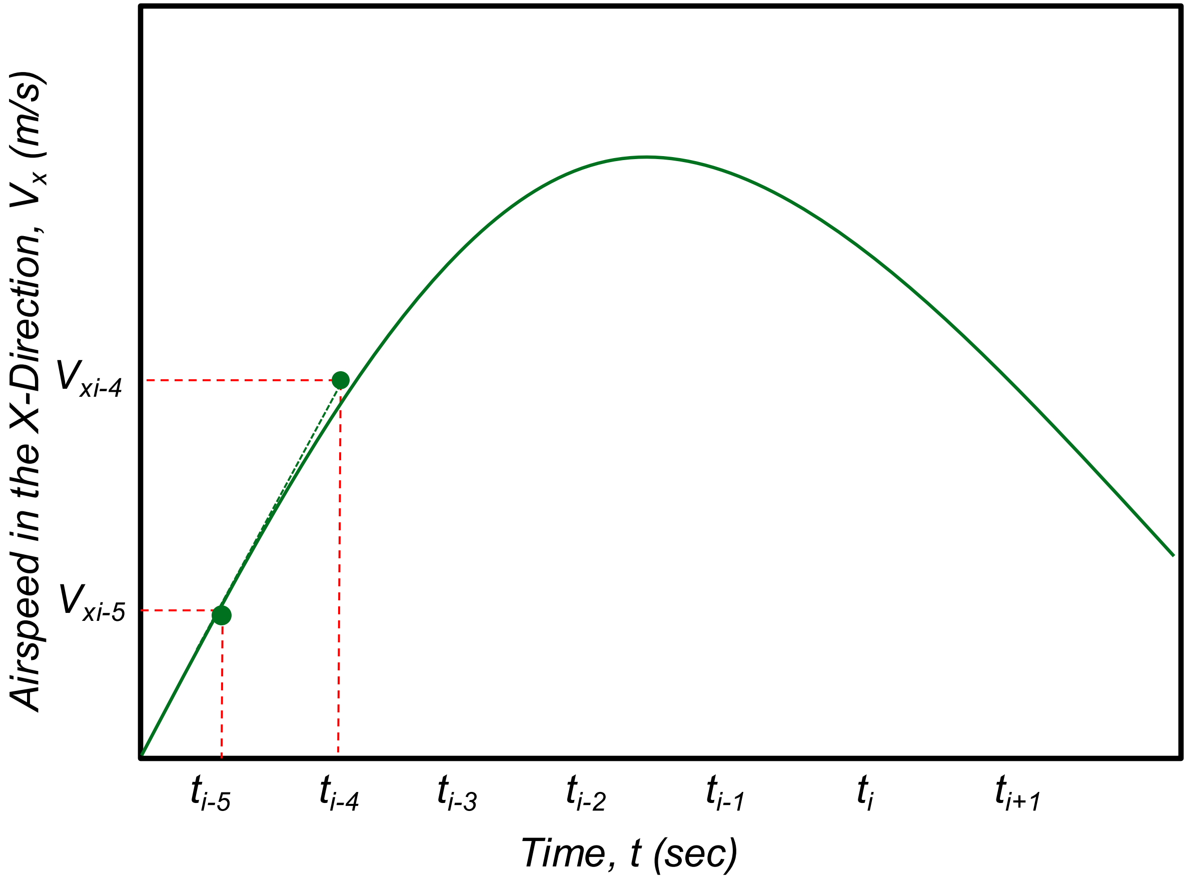
4th Order (Padé) approximations for V_{i+1}

$$\frac{dV_{xi}}{dt} \cong \frac{\Delta V_{xi}}{\Delta t} = \frac{-V_{xi+2} + 8V_{xi+1} - 8V_{xi-1} + V_{xi-2}}{12\Delta t} \quad \therefore \quad V_{xi+1} = +8V_{xi} - 8V_{xi-2} + V_{xi-3} - 12\Delta t \left.\frac{dV_x}{dt}\right|_i$$



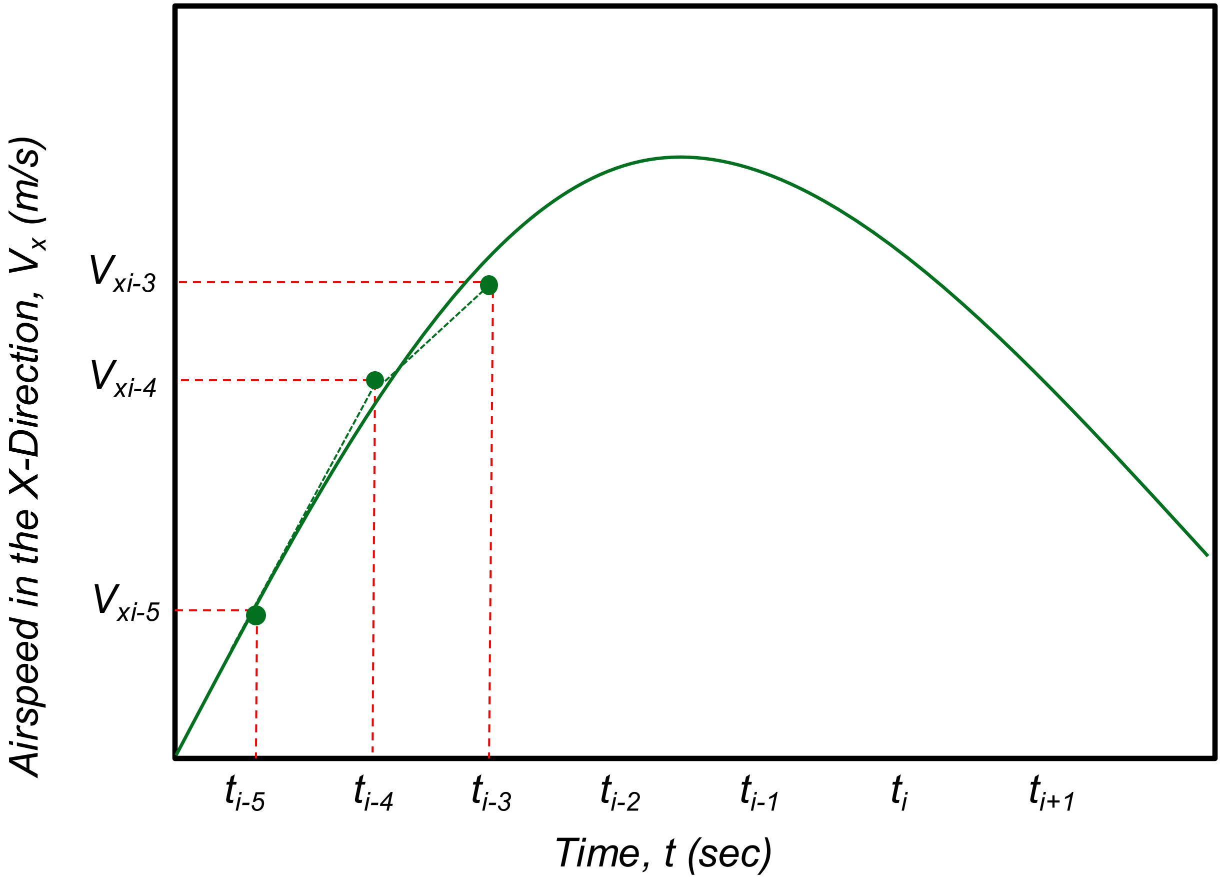
1st Order Proverse Engineering

Problems with Numerical Modeling



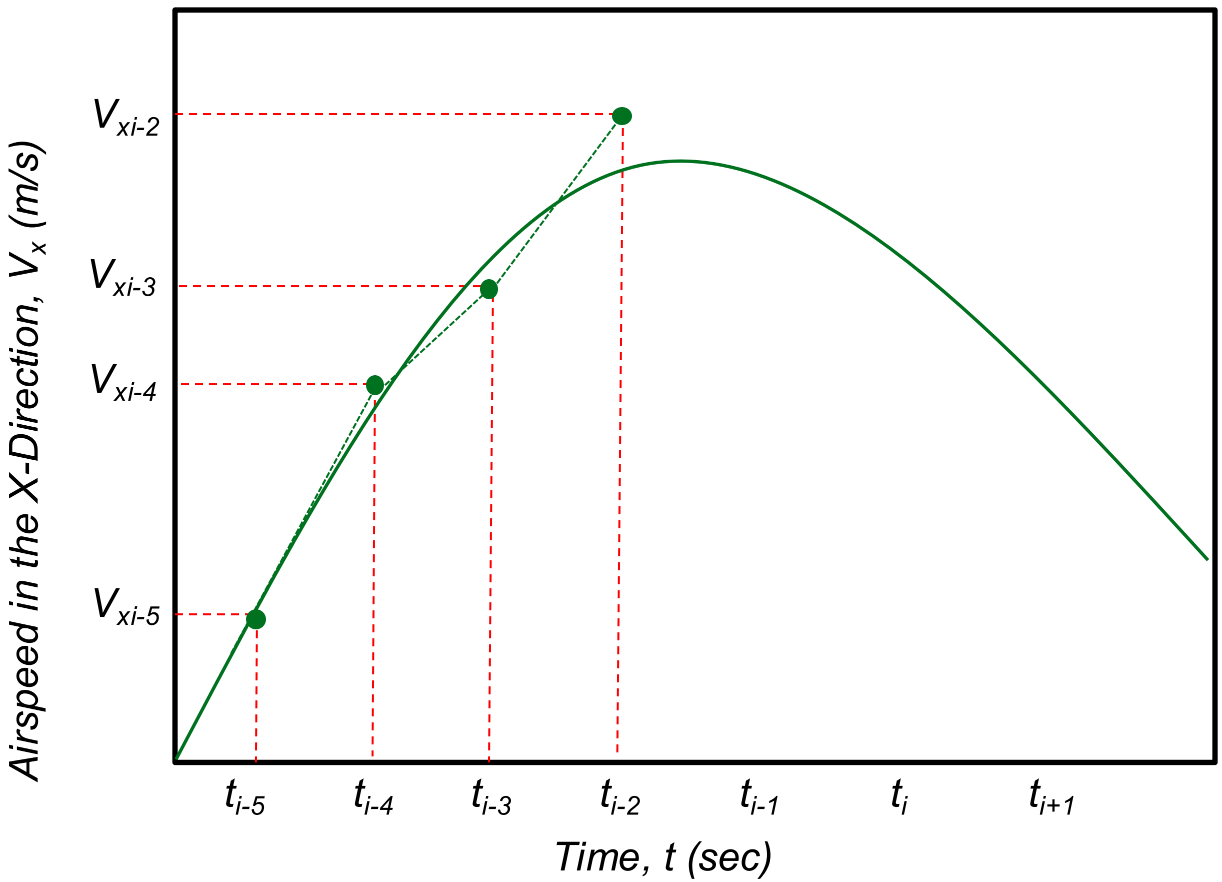
1st Order Proverse Engineering

Problems with Numerical Modeling



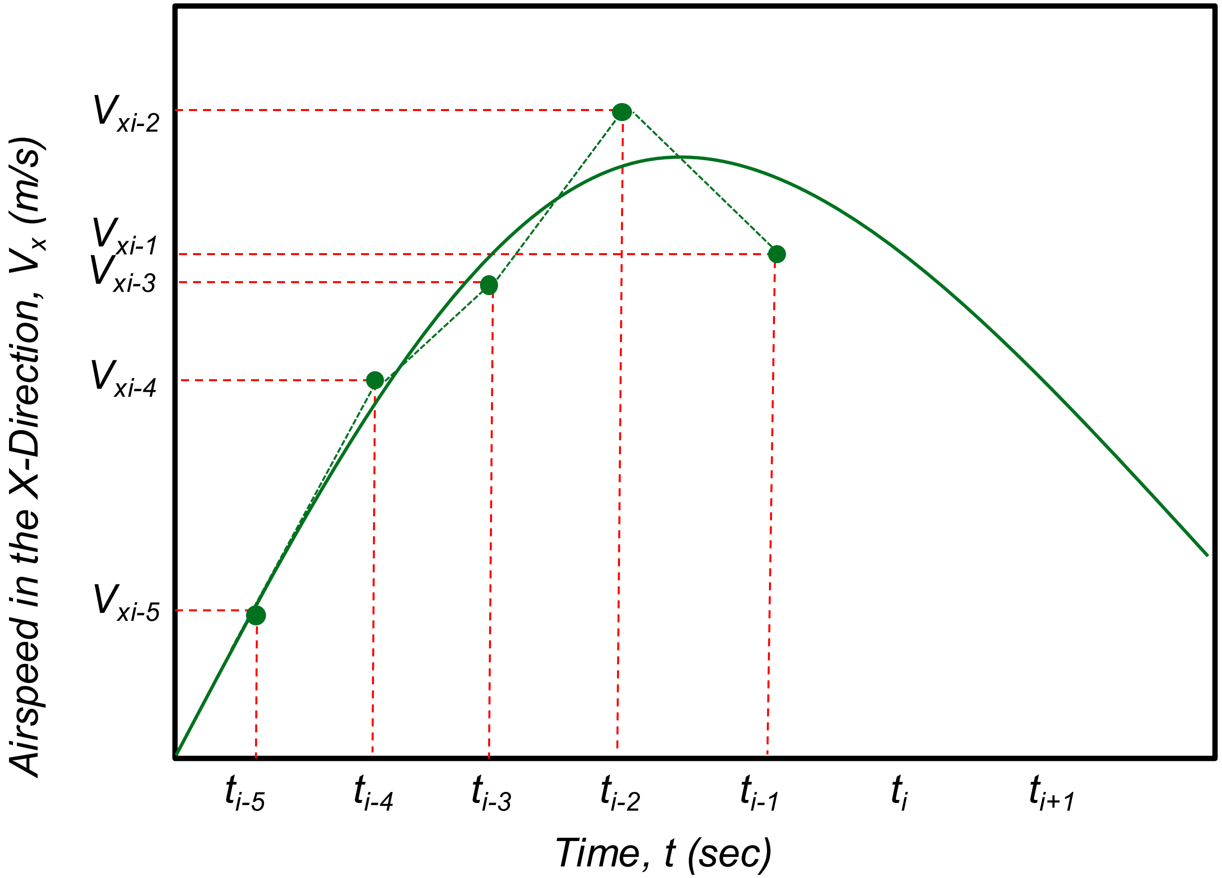
1st Order Proverse Engineering

Problems with Numerical Modeling



1st Order Proverse Engineering

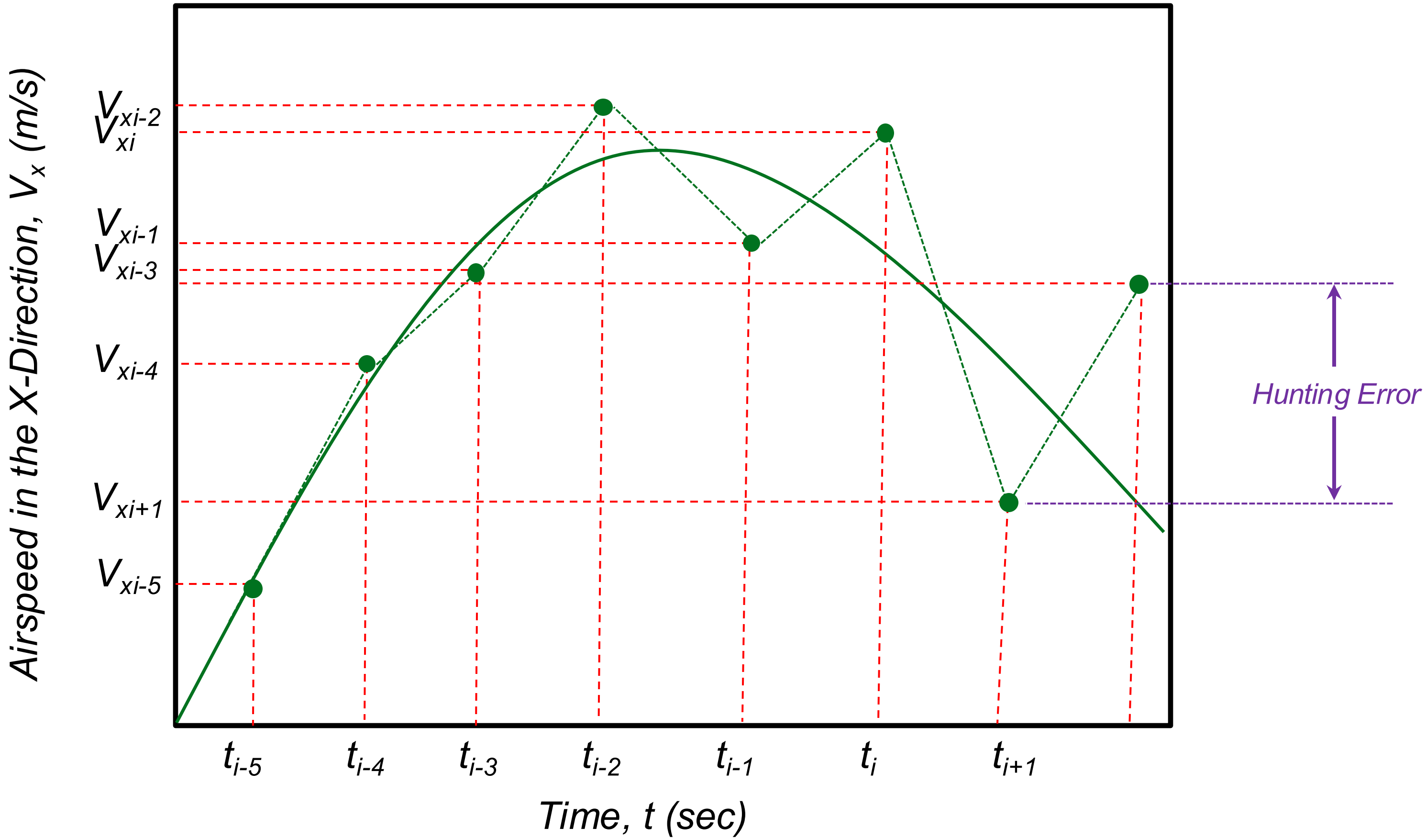
Problems with Numerical Modeling



1st Order Proverse Engineering

Problems with Numerical Modeling

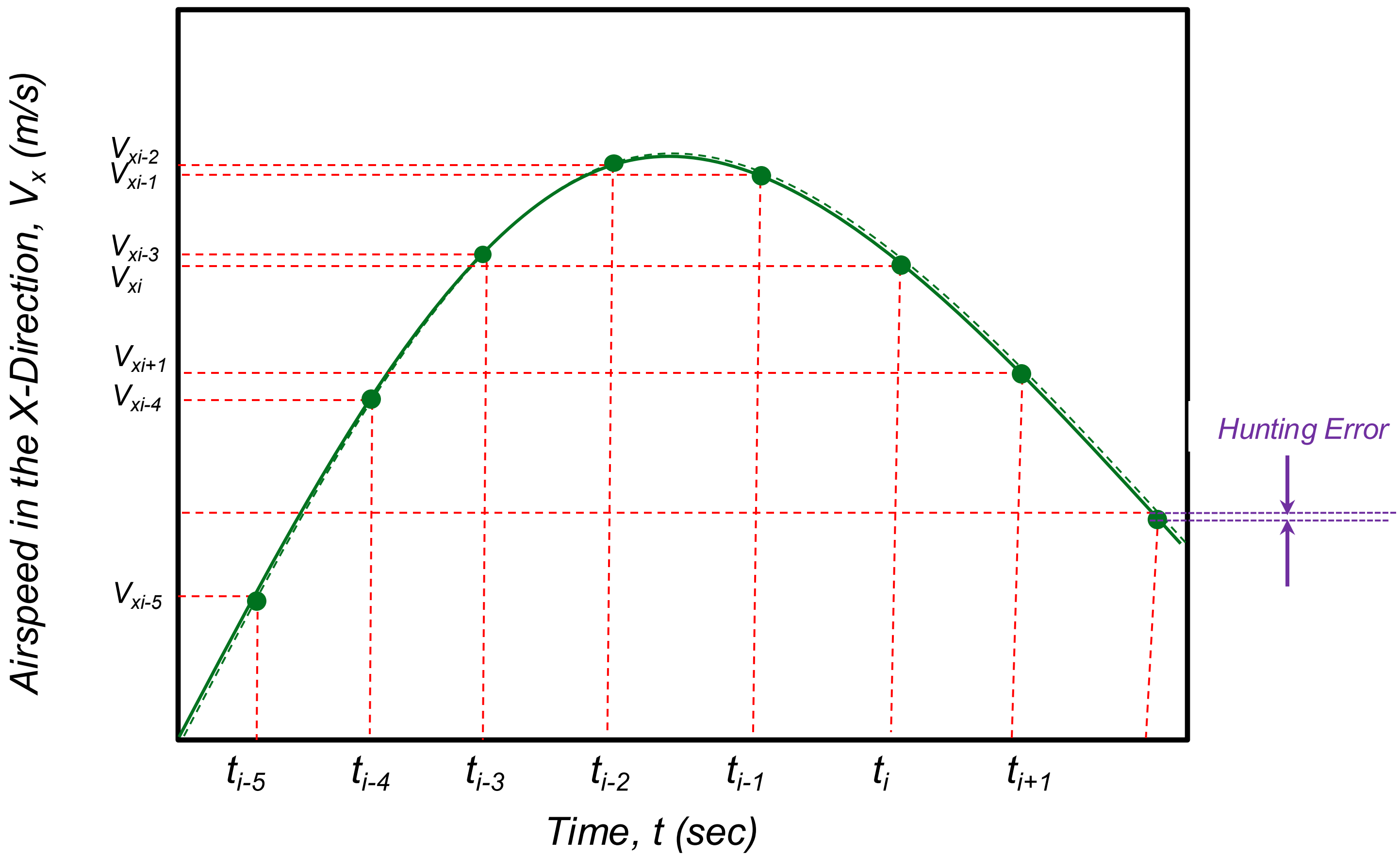
Numerical Instability: Steps too coarse for given function, order of estimation too low



1st Order Proverse Engineering

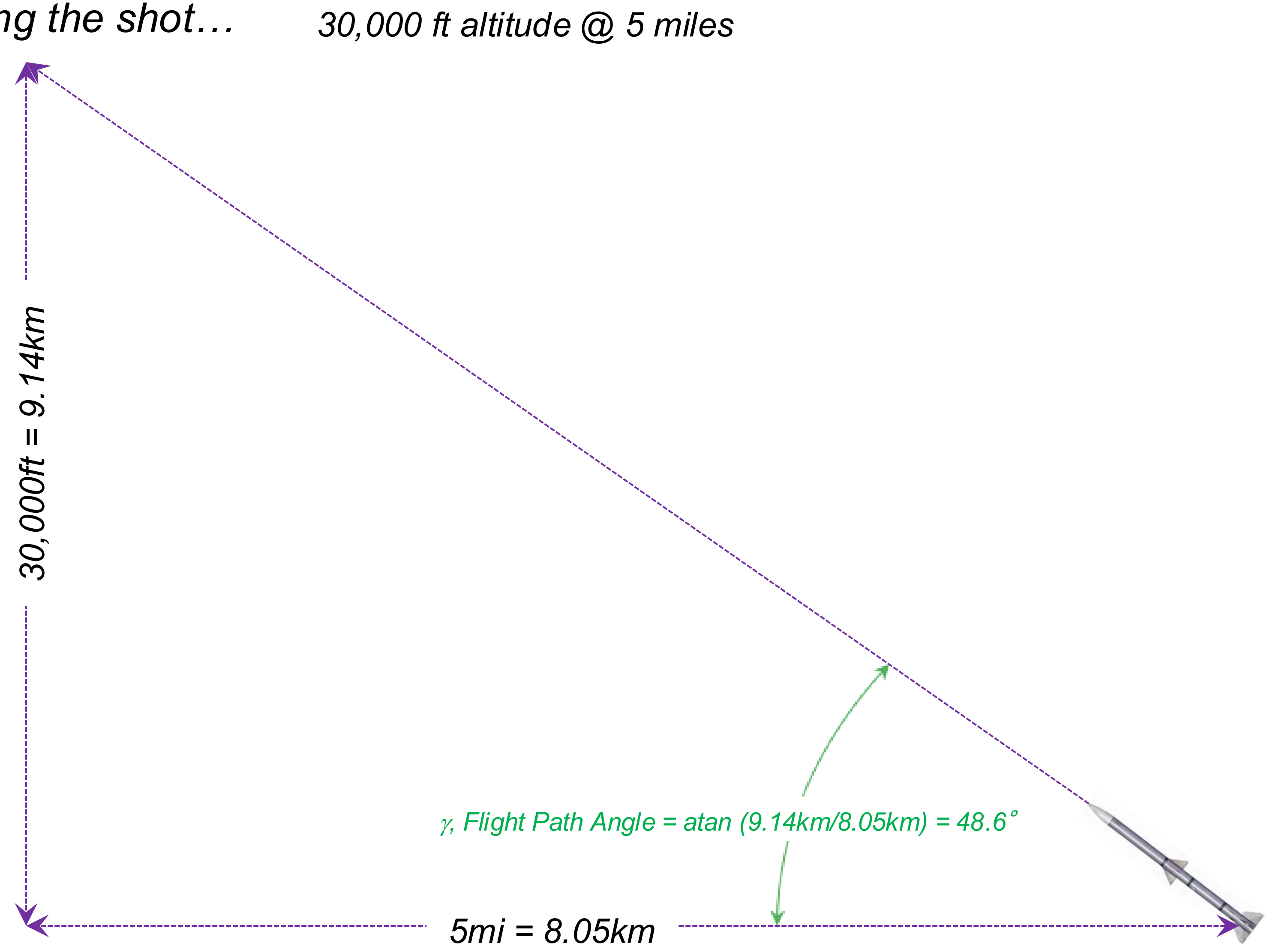
High Order Estimates

High order estimates allow coarser time steps with lower errors (faster codes)



1st Order Proverse Engineering

Coding the shot…



1st Order Proverse Engineering

Coding the shot…

γ_i appears multiple times in eq. 5 & 6… and is extremely important

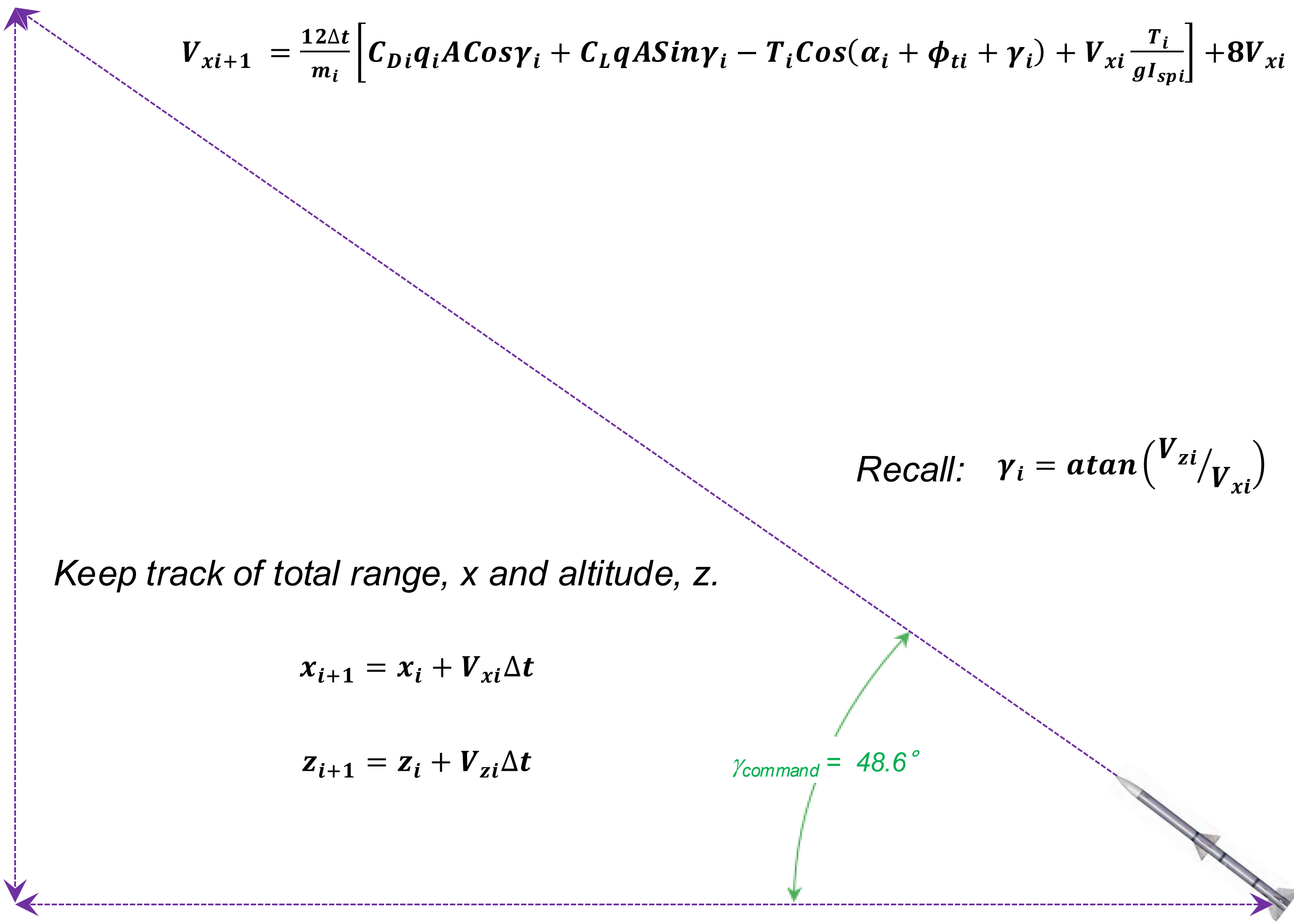
$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{gI_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

Recall: $\gamma_i = atan\left({}^{V_{zi}}/_{V_{xi}}\right)$

Keep track of total range, x and altitude, z.

$$x_{i+1} = x_i + V_{xi}\Delta t$$

$$z_{i+1} = z_i + V_{zi}\Delta t$$

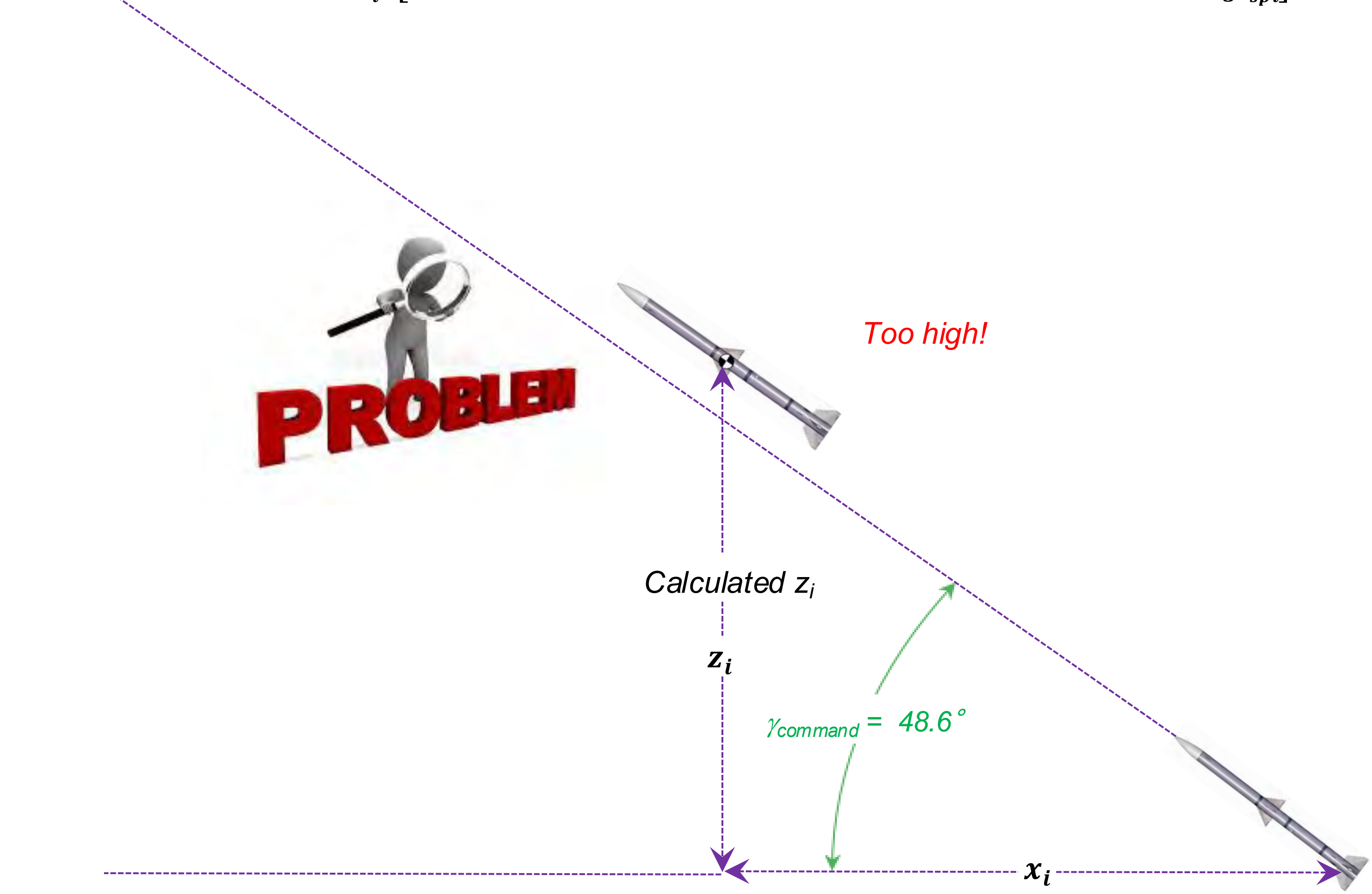


1st Order Proverse Engineering

Coding the shot…

Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di}q_iACos\gamma_i + C_LqASin\gamma_i - T_iCos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{gI_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

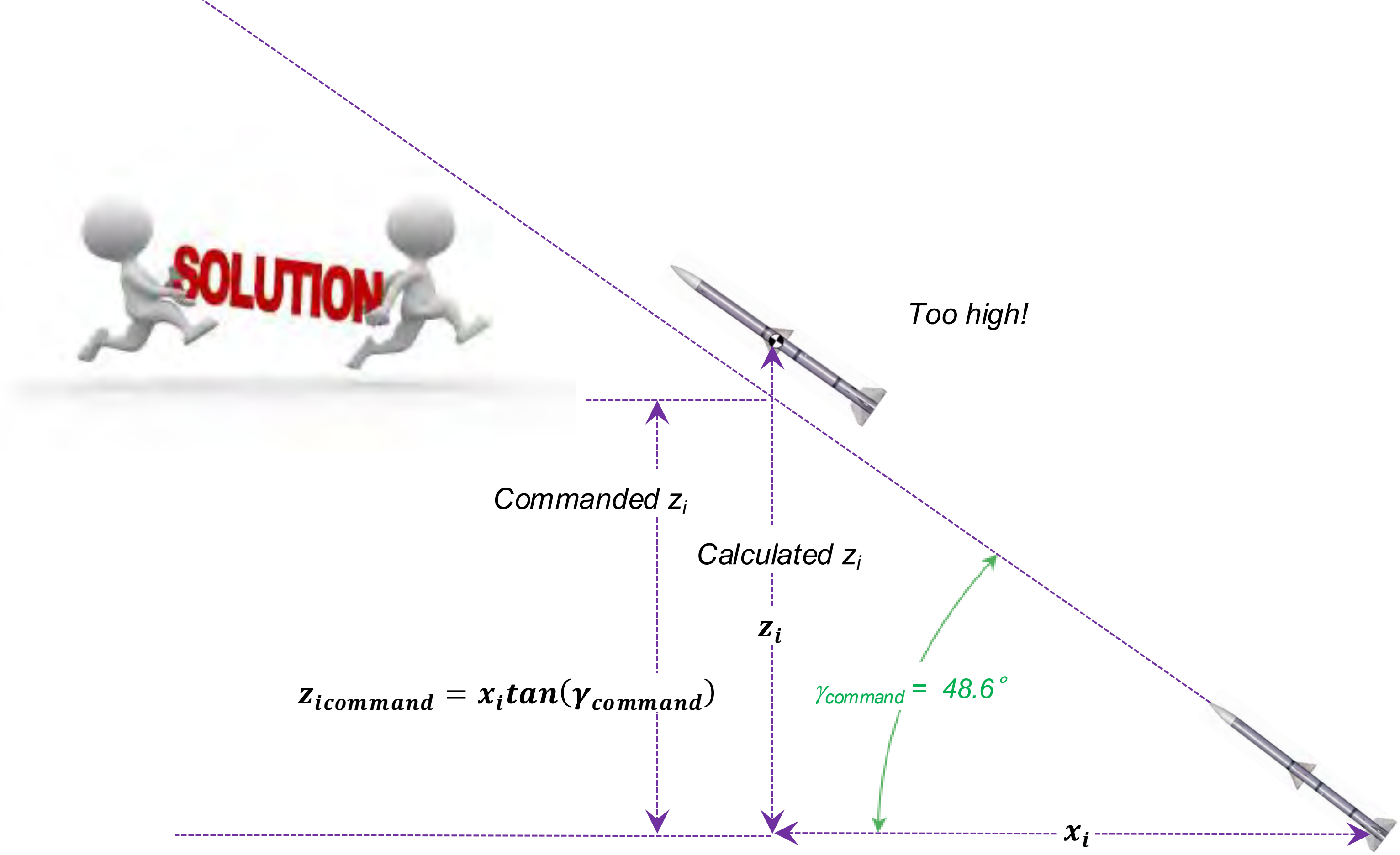


1st Order Proverse Engineering

Coding the shot…

Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{g I_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$



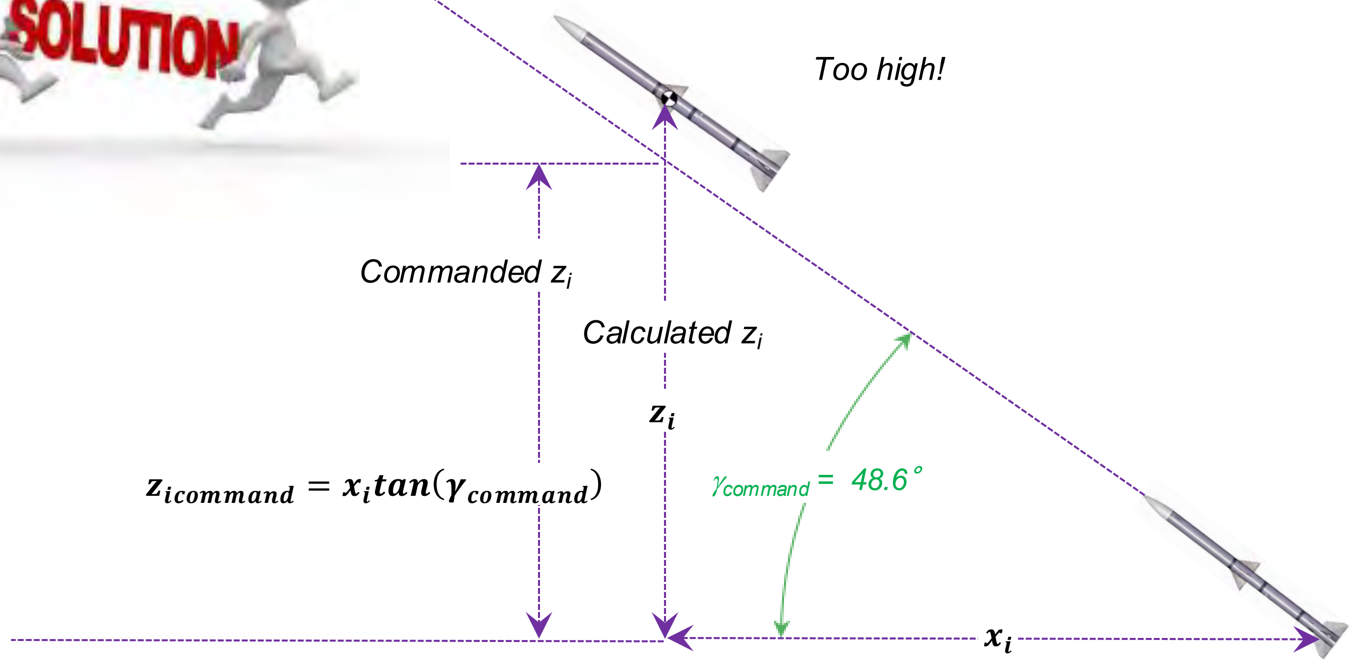
1st Order Proverse Engineering

Coding the shot…

Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{g I_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

Cut altitude to intercept line by reducing α_{i+1}



1st Order Proverse Engineering

Coding the shot…

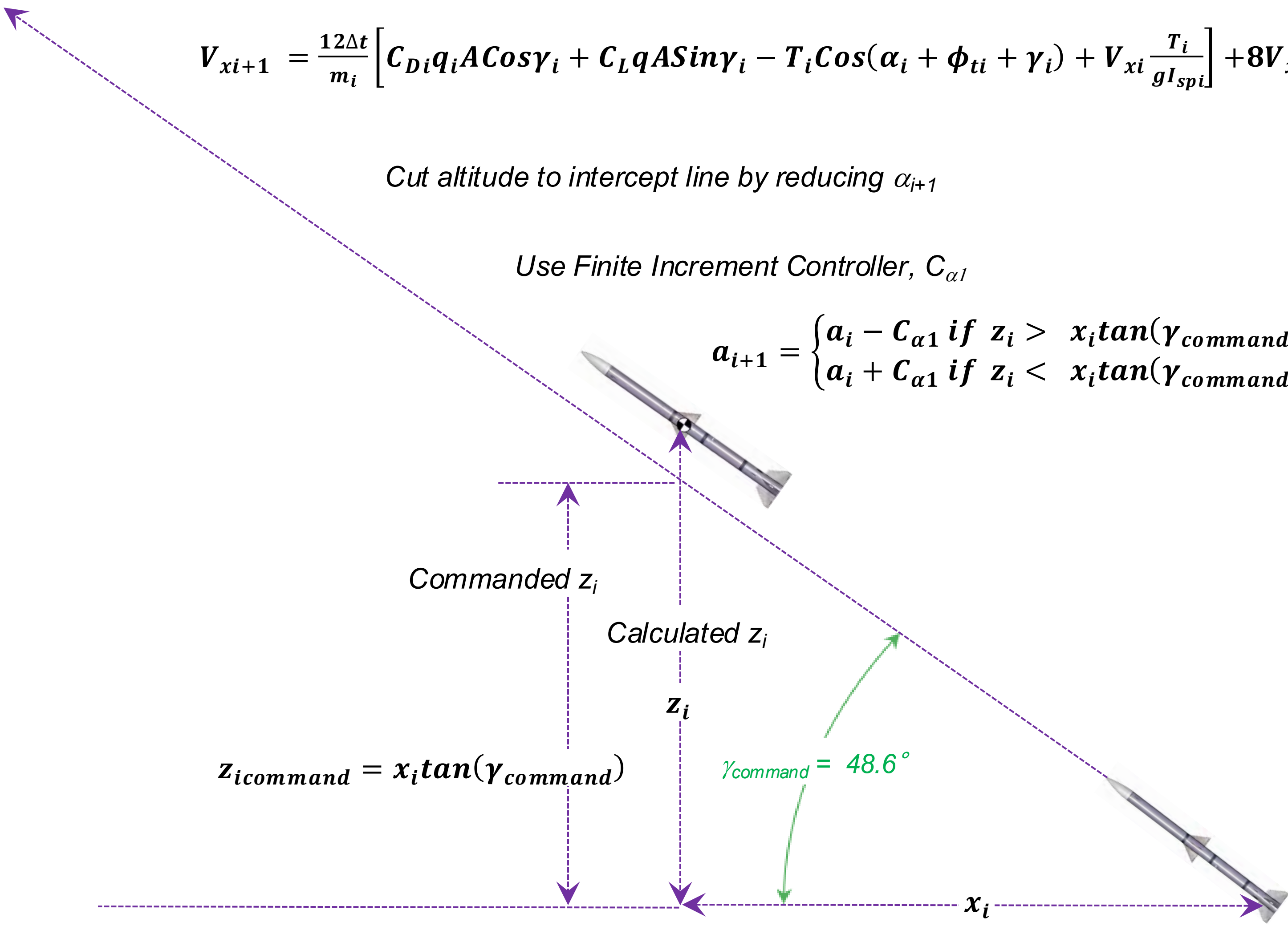
Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di}q_iACos\gamma_i + C_LqASin\gamma_i - T_iCos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{gI_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

Cut altitude to intercept line by reducing α_{i+1}

Use Finite Increment Controller, $C_{\alpha 1}$

$$a_{i+1} = \begin{cases} a_i - C_{\alpha 1}\ if\ z_i > x_i tan(\gamma_{command}) \\ a_i + C_{\alpha 1}\ if\ z_i < x_i tan(\gamma_{command}) \end{cases}$$



1st Order Proverse Engineering

Coding the shot…

Update the angle of attack to fly on trajectory:

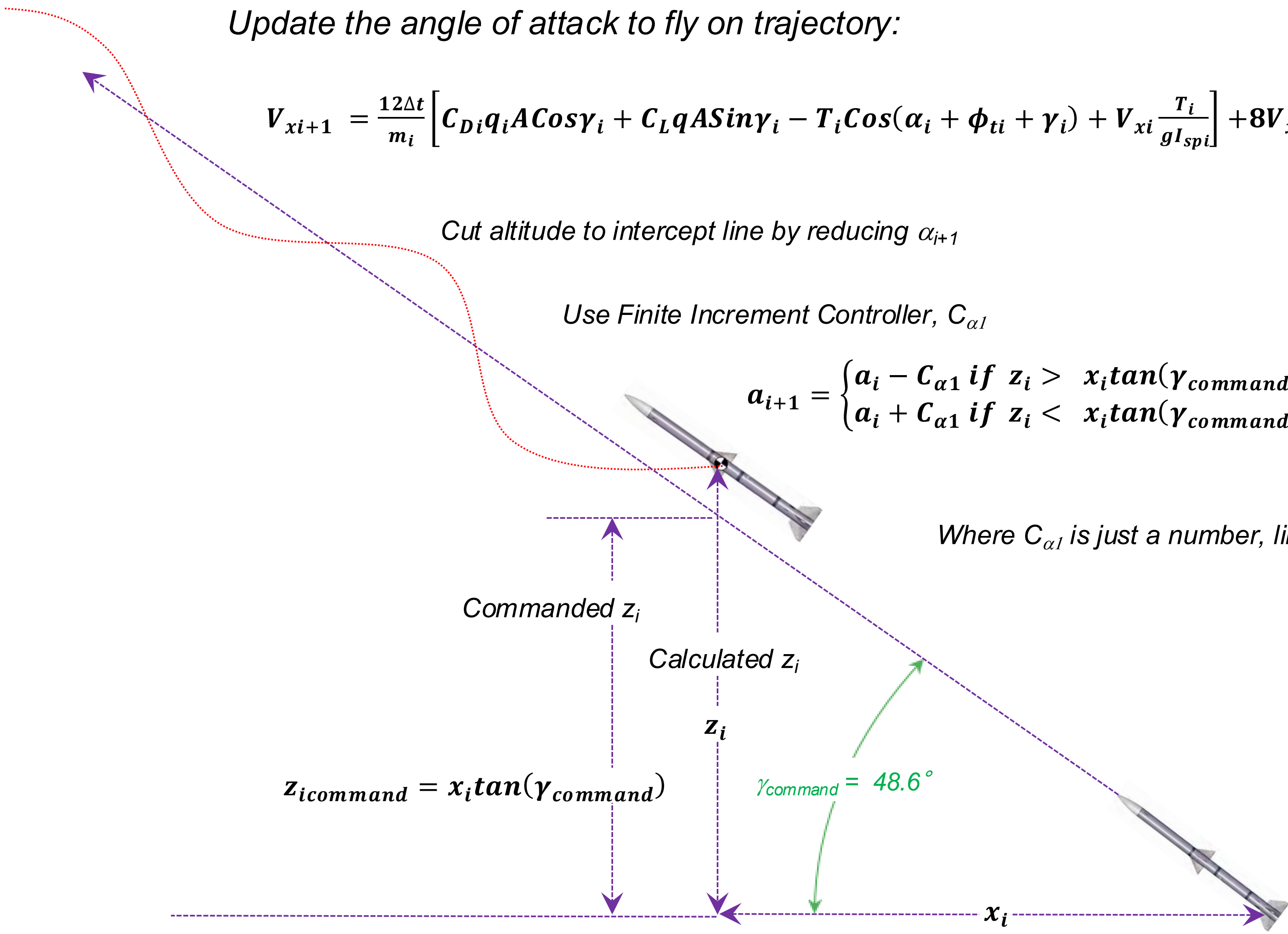
$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{g I_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

Cut altitude to intercept line by reducing α_{i+1}

Use Finite Increment Controller, $C_{\alpha 1}$

$$a_{i+1} = \begin{cases} a_i - C_{\alpha 1} \; if \;\; z_i > \;\; x_i tan(\gamma_{command}) \\ a_i + C_{\alpha 1} \; if \;\; z_i < \;\; x_i tan(\gamma_{command}) \end{cases}$$

Where $C_{\alpha 1}$ is just a number, like 0.1 deg.



1st Order Proverse Engineering

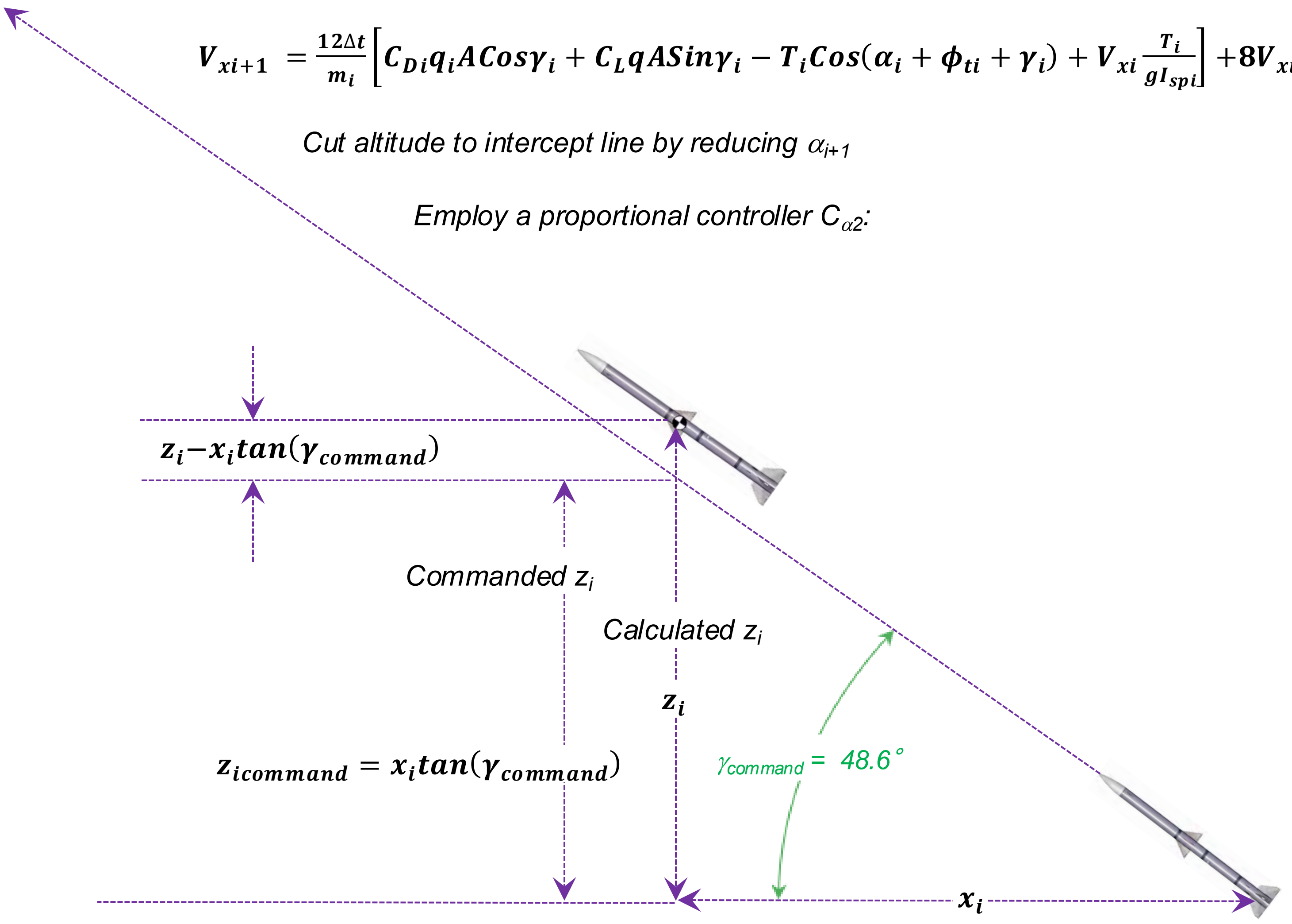
Coding the shot…

Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di}q_iACos\gamma_i + C_LqASin\gamma_i - T_iCos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{gI_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

Cut altitude to intercept line by reducing α_{i+1}

Employ a proportional controller $C_{\alpha 2}$:



1st Order Proverse Engineering

Coding the shot…

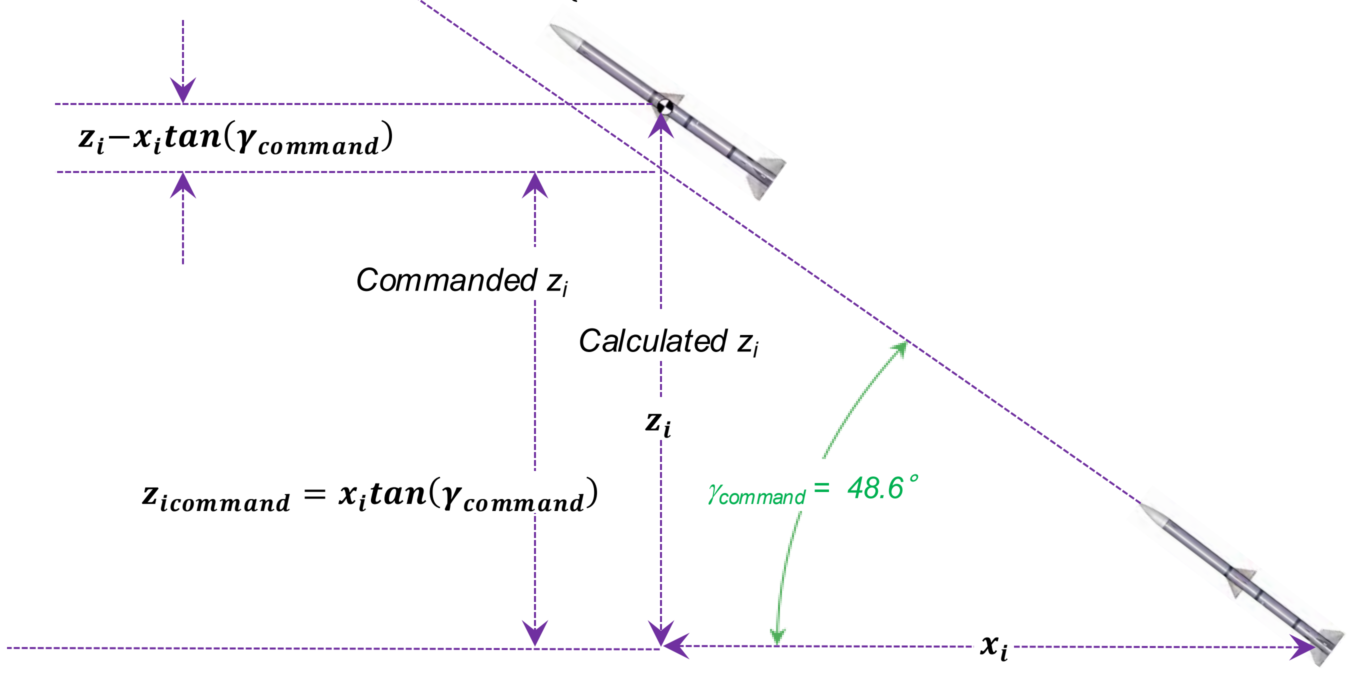
Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{g I_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

Cut altitude to intercept line by reducing α_{i+1}

Employ a proportional controller $C_{\alpha 2}$:

$$a_{i+1} = \begin{cases} a_i - C_{\alpha 1} - C_{\alpha 2}[\ z_i - x_i tan(\gamma_{command})]\ if\ z_i > x_i tan(\gamma_{command}) \\ a_i + C_{\alpha 1} + C_{\alpha 2}[\ z_i - x_i tan(\gamma_{command})]\ if\ z_i < x_i tan(\gamma_{command}) \end{cases}$$



1st Order Proverse Engineering

Coding the shot…

Update the angle of attack to fly on trajectory:

$$V_{xi+1} = \frac{12\Delta t}{m_i}\left[C_{Di} q_i A Cos\gamma_i + C_L q A Sin\gamma_i - T_i Cos(\alpha_i + \phi_{ti} + \gamma_i) + V_{xi}\frac{T_i}{g I_{spi}}\right] + 8V_{xi} - 8V_{xi-2} + V_{xi-3}$$

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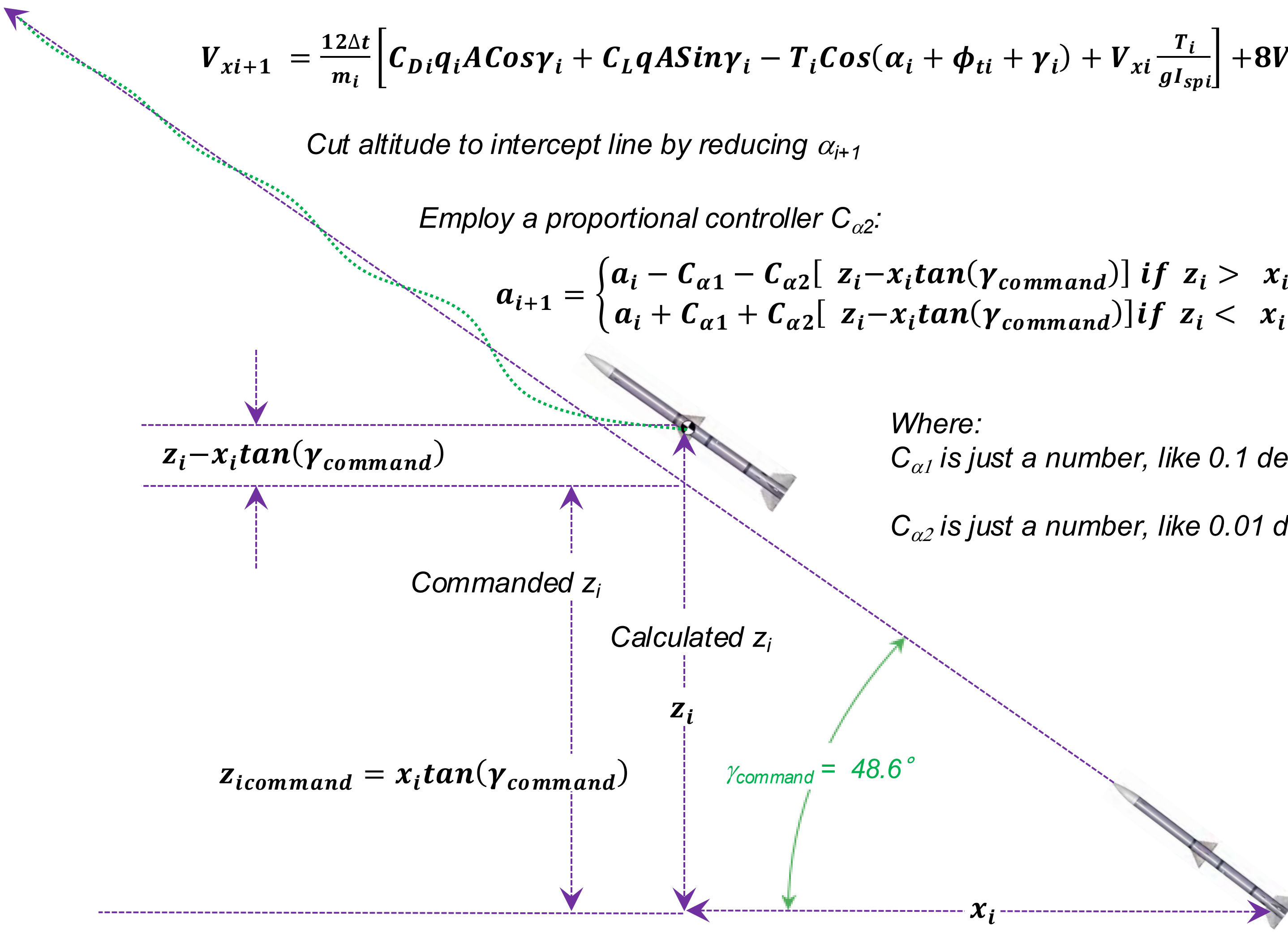


Where:

$C_{\alpha 1}$ is just a number, like 0.1 deg.

$C_{\alpha 2}$ is just a number, like 0.01 deg./m

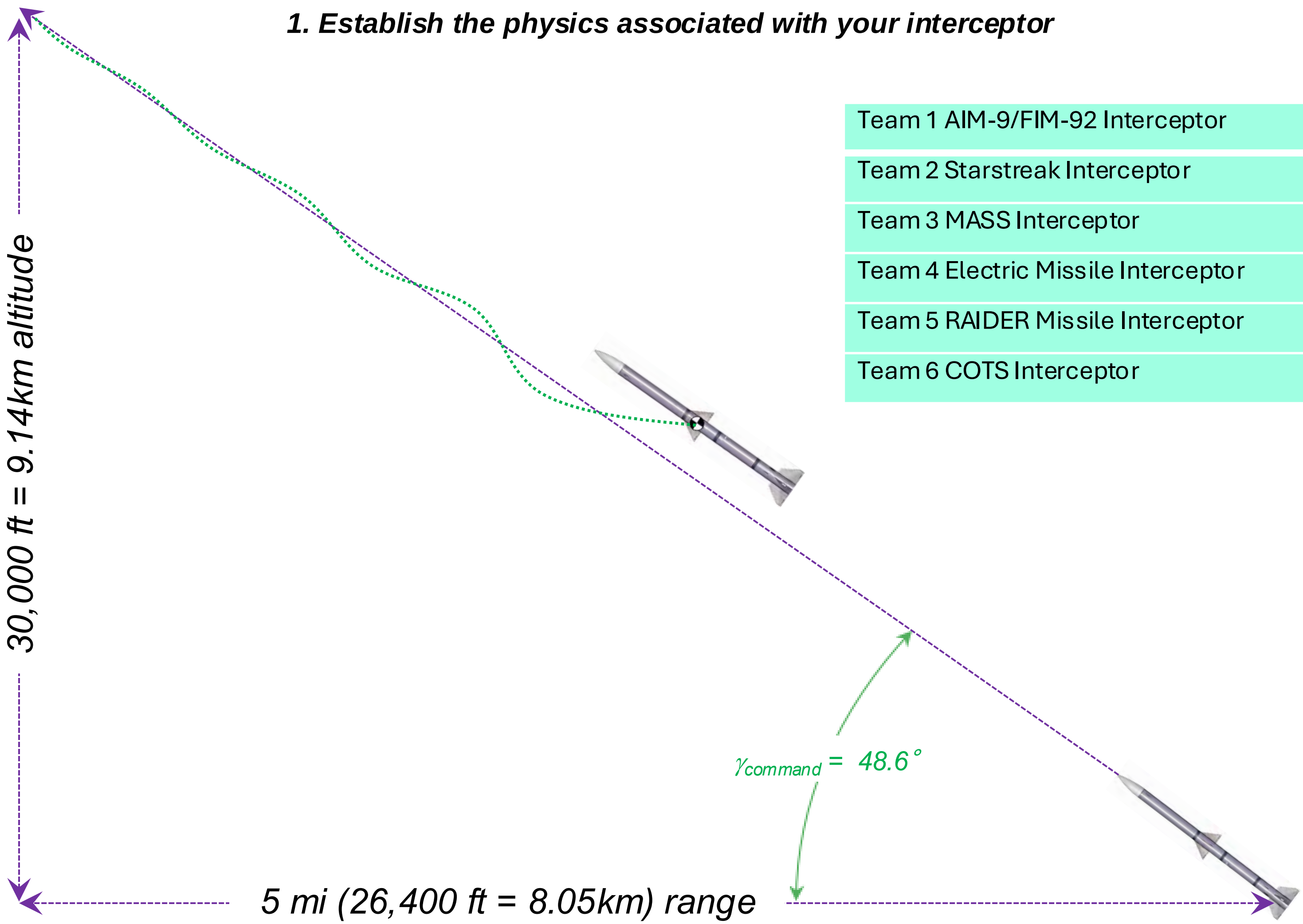
1st Order Proverse Engineering

Step back and examine the mission:

Proverse design a missile to make the shot

1. Establish the physics associated with your interceptor

Team 1 AIM-9/FIM-92 Interceptor
Team 2 Starstreak Interceptor
Team 3 MASS Interceptor
Team 4 Electric Missile Interceptor
Team 5 RAIDER Missile Interceptor
Team 6 COTS Interceptor

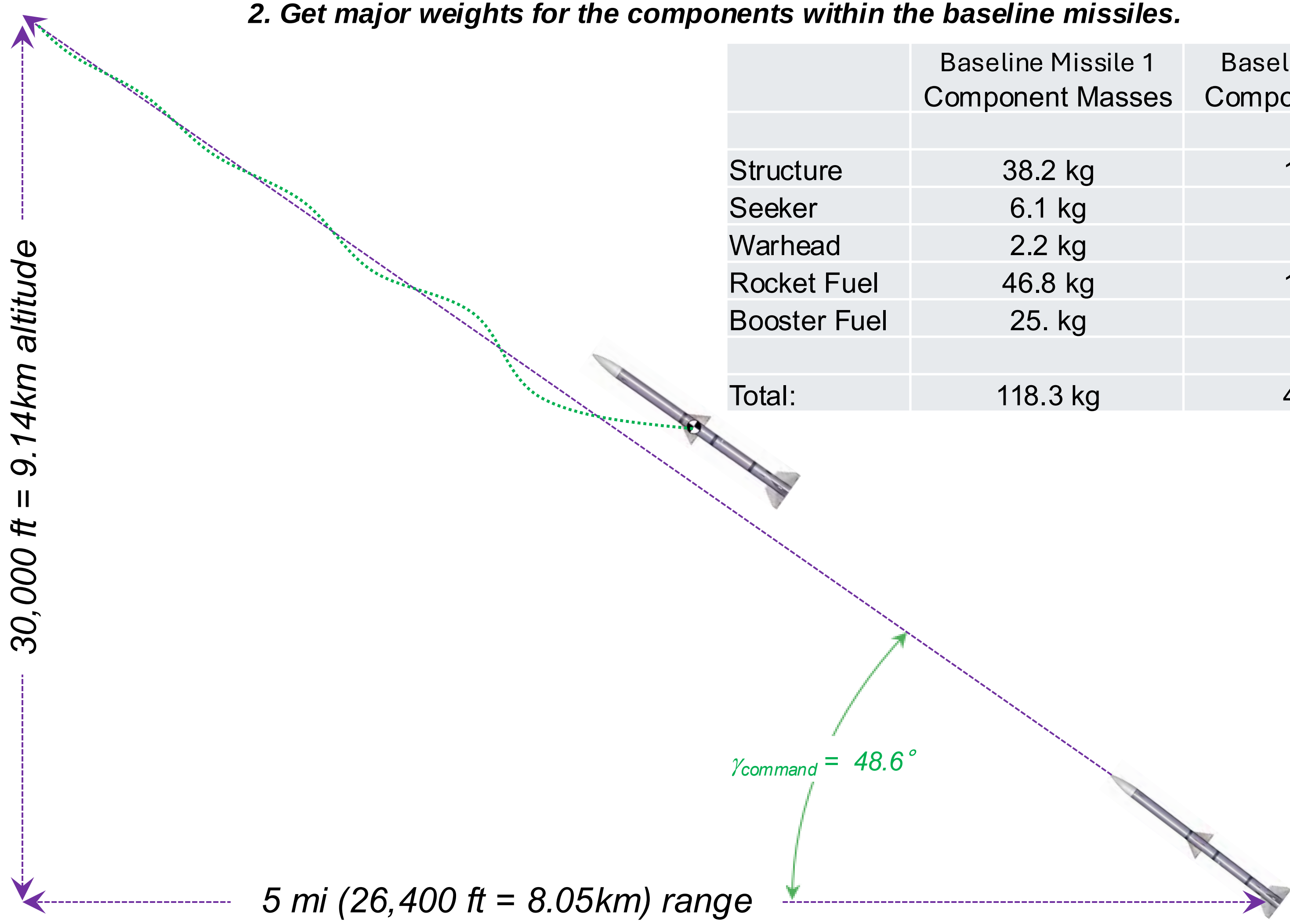


1st Order Proverse Engineering

Proverse design a missile to make the shot

2. Get major weights for the components within the baseline missiles.

	Baseline Missile 1 Component Masses	Baseline Missile 2 Component Masses
Structure	38.2 kg	16.5 kg
Seeker	6.1 kg	3.8 kg
Warhead	2.2 kg	1.5 kg
Rocket Fuel	46.8 kg	18.3 kg
Booster Fuel	25. kg	7.2 kg
Total:	118.3 kg	47.3 kg



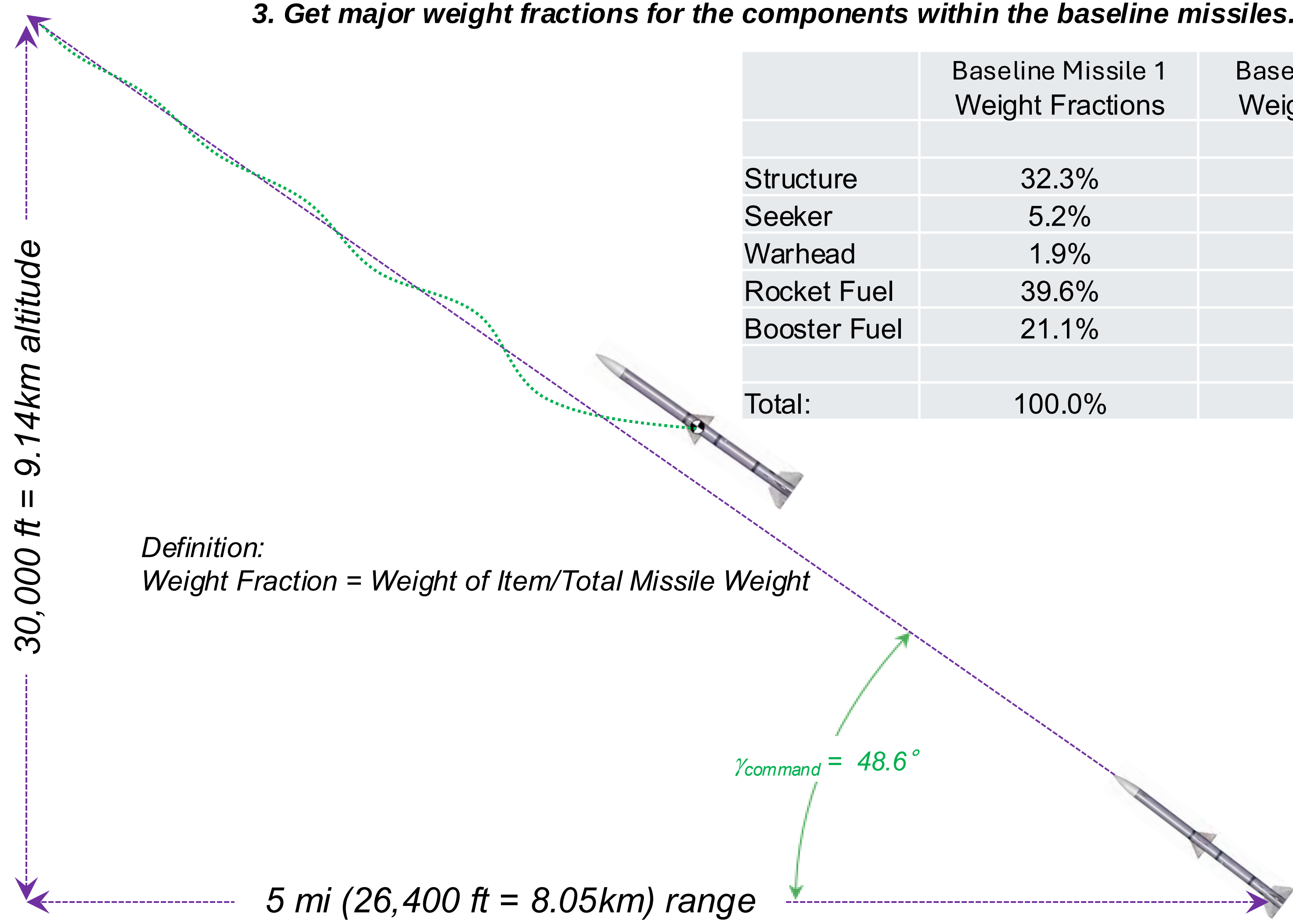
1st Order Proverse Engineering

Proverse design a missile to make the shot

3. Get major weight fractions for the components within the baseline missiles.

	Baseline Missile 1 Weight Fractions	Baseline Missile 2 Weight Fractions
Structure	32.3%	34.9%
Seeker	5.2%	8.0%
Warhead	1.9%	3.2%
Rocket Fuel	39.6%	38.7%
Booster Fuel	21.1%	15.2%
Total:	100.0%	100.0%

Definition:
Weight Fraction = Weight of Item/Total Missile Weight



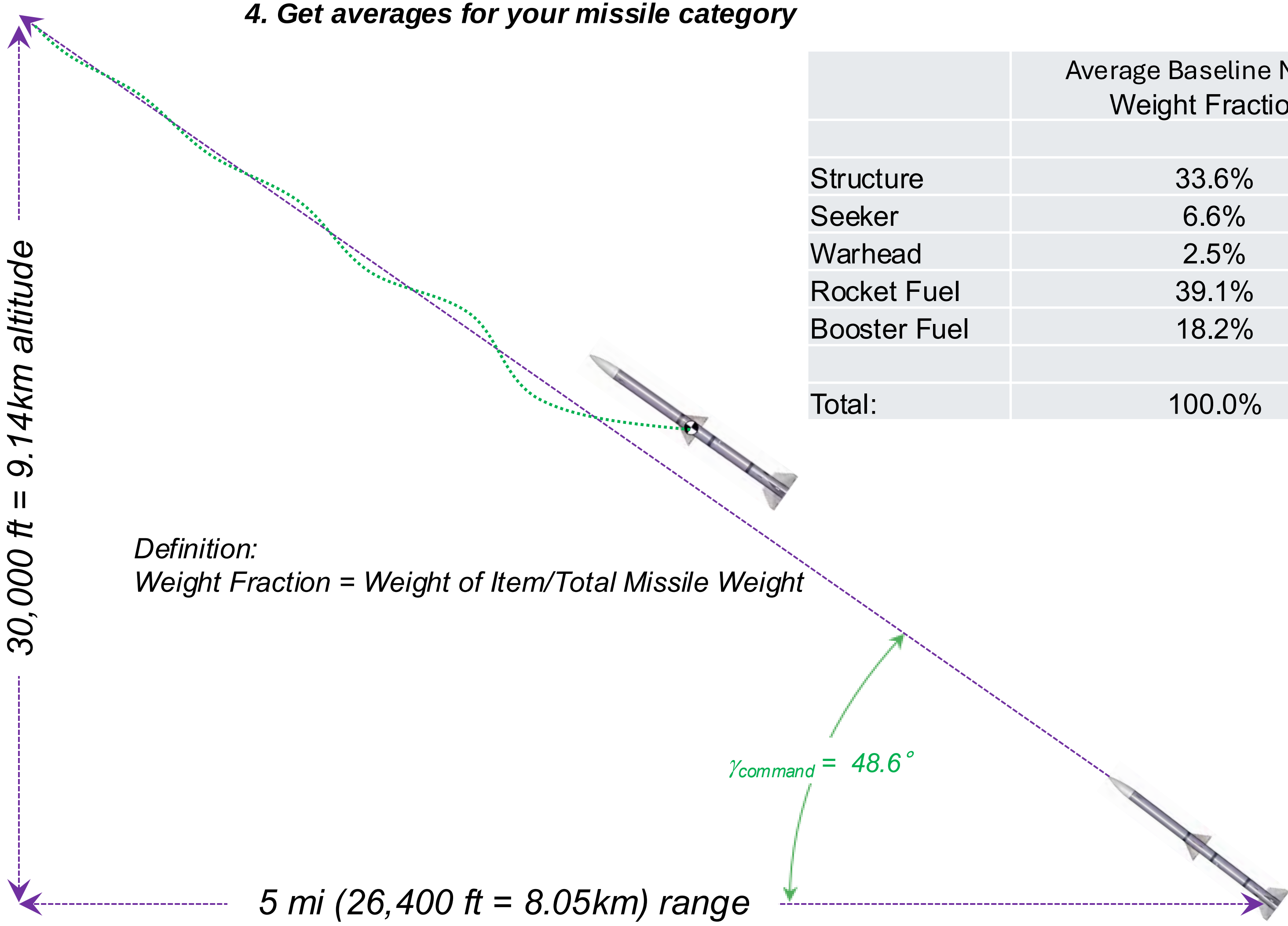
1st Order Proverse Engineering

Proverse design a missile to make the shot

4. Get averages for your missile category

	Average Baseline Missile Weight Fractions
Structure	33.6%
Seeker	6.6%
Warhead	2.5%
Rocket Fuel	39.1%
Booster Fuel	18.2%
Total:	100.0%

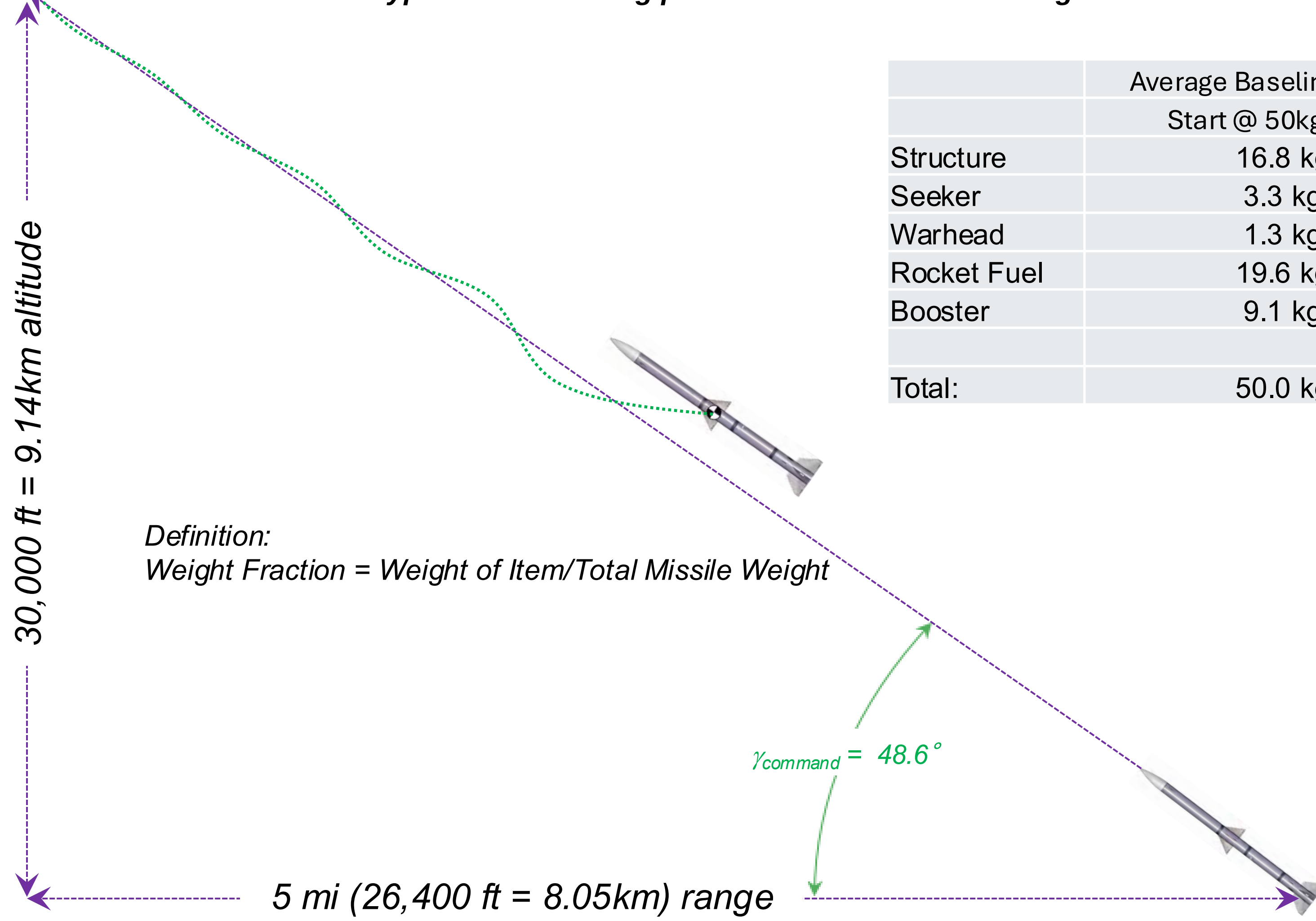
Definition:
Weight Fraction = Weight of Item/Total Missile Weight



1st Order Proverse Engineering

Proverse design a missile to make the shot

5. Make hypothetical starting point missile based on average baseline missile



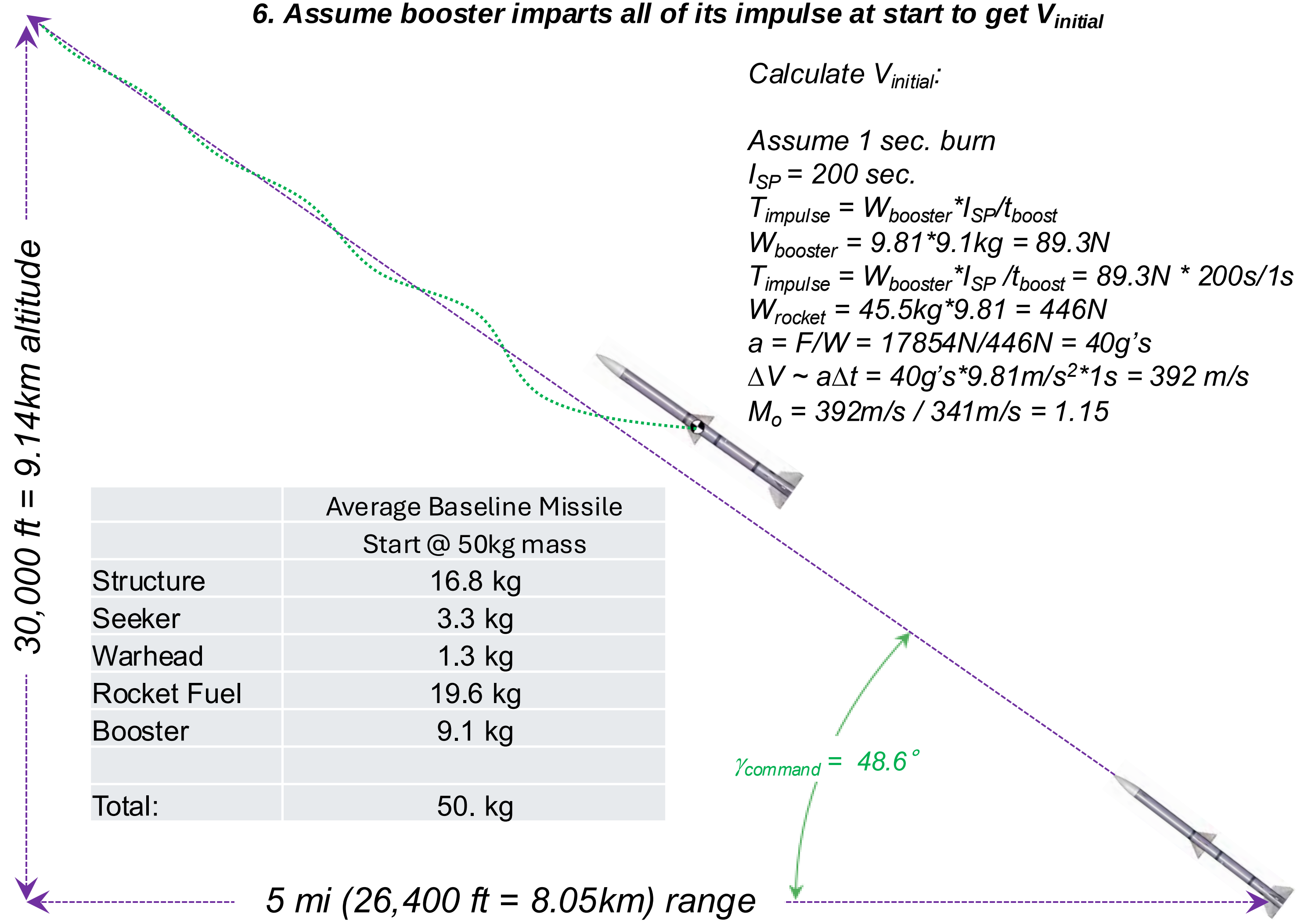
	Average Baseline Missile
	Start @ 50kg mass
Structure	16.8 kg
Seeker	3.3 kg
Warhead	1.3 kg
Rocket Fuel	19.6 kg
Booster	9.1 kg
Total:	50.0 kg

Definition:
Weight Fraction = Weight of Item/Total Missile Weight

1st Order Proverse Engineering

Proverse design a missile to make the shot

6. Assume booster imparts all of its impulse at start to get $V_{initial}$



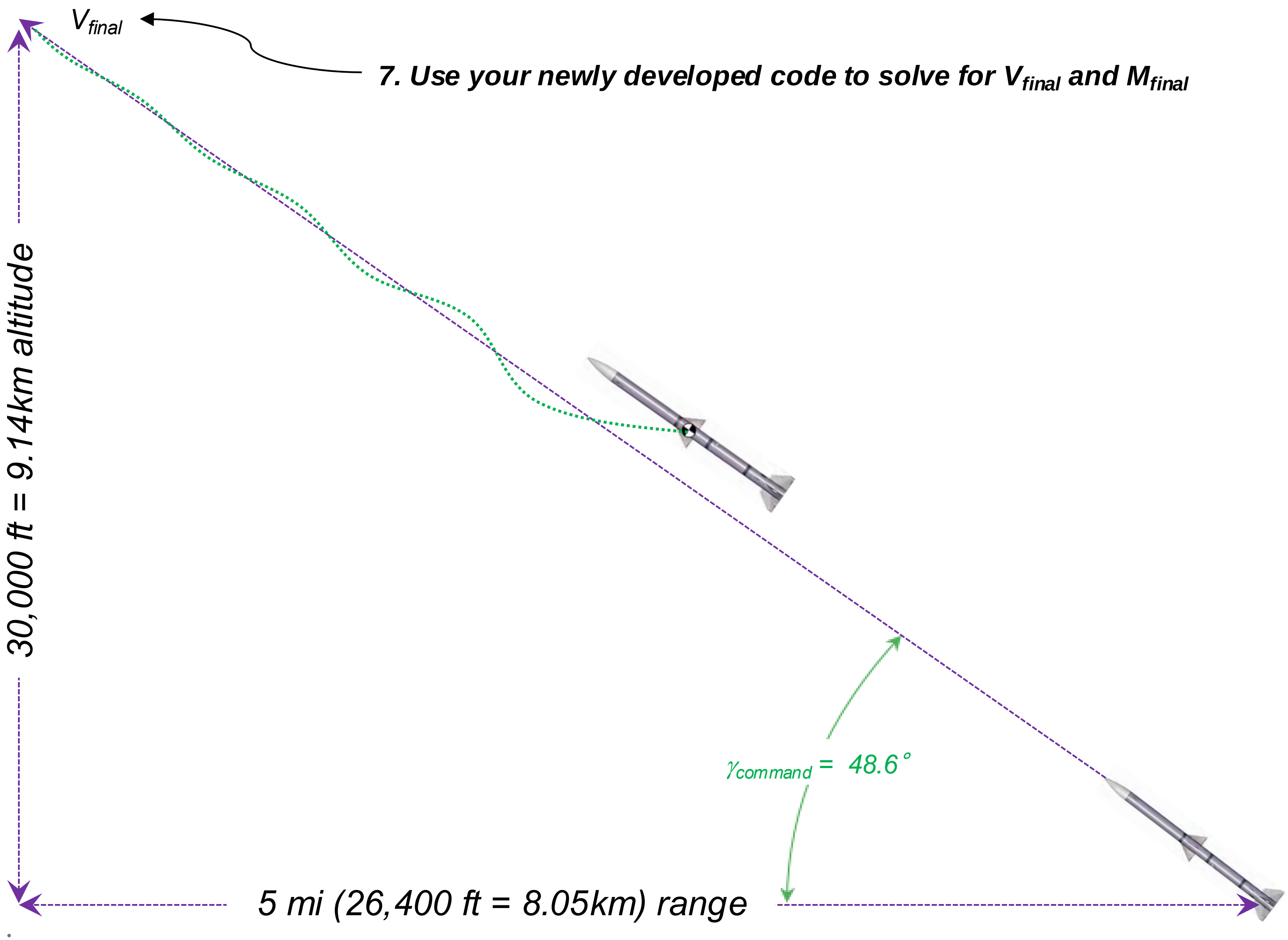
Calculate $V_{initial}$:

Assume 1 sec. burn
$I_{SP} = 200$ sec.
$T_{impulse} = W_{booster} * I_{SP} / t_{boost}$
$W_{booster} = 9.81 * 9.1kg = 89.3N$
$T_{impulse} = W_{booster} * I_{SP} / t_{boost} = 89.3N * 200s/1s = 17854N$
$W_{rocket} = 45.5kg * 9.81 = 446N$
$a = F/W = 17854N/446N = 40g's$
$\Delta V \sim a\Delta t = 40g's * 9.81m/s^2 * 1s = 392$ m/s
$M_o = 392m/s / 341m/s = 1.15$

	Average Baseline Missile
	Start @ 50kg mass
Structure	16.8 kg
Seeker	3.3 kg
Warhead	1.3 kg
Rocket Fuel	19.6 kg
Booster	9.1 kg
Total:	50. kg

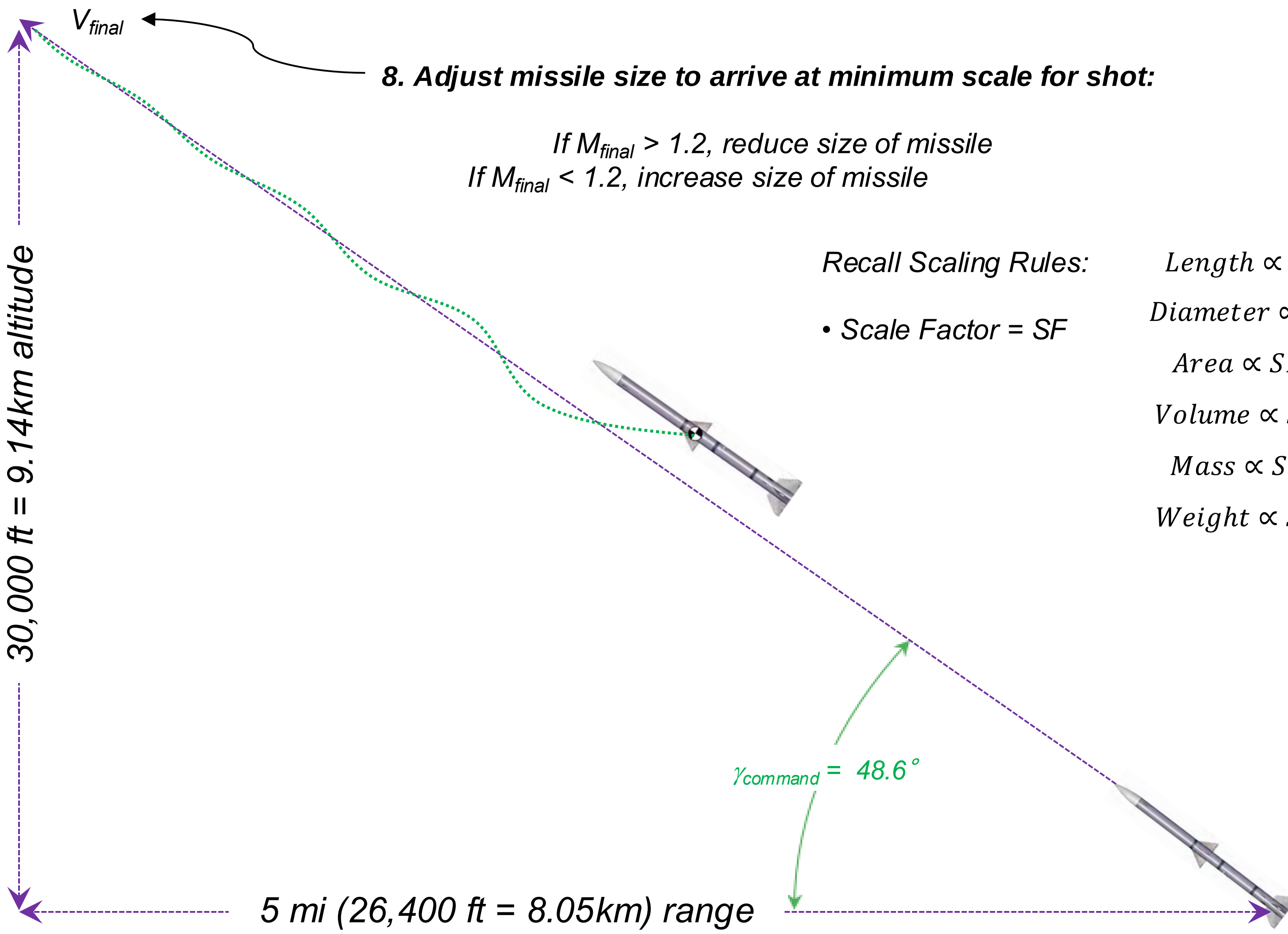
1st Order Proverse Engineering

Proverse design a missile to make the shot



1st Order Proverse Engineering

Proverse design a missile to make the shot



8. Adjust missile size to arrive at minimum scale for shot:

If $M_{final} > 1.2$, reduce size of missile
If $M_{final} < 1.2$, increase size of missile

Recall Scaling Rules:

- *Scale Factor = SF*

$Length \propto SF$

$Diameter \propto SF$

$Area \propto SF^2$

$Volume \propto SF^3$

$Mass \propto SF^3$

$Weight \propto SF^3$

1st Order Proverse Engineering

Proverse design a missile to make the shot

9. Design the desired missile for the shot and CAD all components

1st Order Proverse Engineering

Proverse Design the Display Model for the Air Armament Symposium

10. Approximate Missile Dimensions for the Model

For Instance:
Sizing yields 11.9cm diameter body x 2.06m length
Call out a 5” body diameter (so we can order it)
If we can order > 72” lengths good… otherwise, set length to 72”
Scale fins proportionally

1st Order Proverse Engineering

Proverse Design the Display Model for the Air Armament Symposium

11. Assemble a Bill of Materials (BOM)

- **Part Number**
- **Name of Part**
- **Material from which part is made**
- **Quantity Required**
- Cost per part each. and/or total
- **Source of Part or Material**
- **Identify each part or material type on assembly view**

1st Order Proverse Engineering

Proverse Design the Display Model for the Air Armament Symposium

11. Assemble a Bill of Materials (BOM)

Part No.	Part Name & Mfg no.	Material	Qty. Rq'd	Cost per Item	Source/ Supplier	Component Weight (lb)	Weight Fraction (%)
1	Nose Cone	Fiberglass-Epoxy	1 ea.	$32 ea	Bill's Rockets bilrock.com	1.1	4.1%
2	Forward body tube	Fiberglass-Epoxy	1 ea.	$28 ea	Bill's Rockets bilrock.com	2.6	9.7%
3	Aft body tube	Fiberglass-Epoxy	1 ea	$28 ea	Bill's Rockets bilrock.com	2.6	9.7%
4	Forward canards	Black aluminum	4 ea.	$5.80 ea	Aerocomposites.com	2.4	9.0%
5	Wings	Black aluminum	4 ea.	$6.72 ea	Aerocomposites.com	3.8	14.2%
6	Fins	Black aluminum	4 ea.	$8.13 ea	Aerocomposites.com	5.6	21.0%
7	Launch Lugs	PLA	2 ea.	~	In-house 3d print	.38	1.4%
8	Booster rocket assembly	PLA	1 ea.	~	In-house 3d print	3.1	11.6%
9	Stand plate	Aluminum	1 ea.	$4.60	Menards	2.3	8.6%
10	Stand flange adapter	Steel	1 ea.	$2.95	Menards	.6	2.2%
11	Stand post	Steel	1 ea	$3.80	Menards	2.2	8.2%
12	Sky Grey Paint	Krylon	1 ea	$5.92	Ace Hardware	~	4.1%
13	Decals	Vinyl	1 pk	$87.60	Decals R Us Decalsrus.com	~	9.7%
total				$104.87		26.6 lb	

1st Order Proverse Engineering

Proverse Design the Display Model for the Air Armament Symposium

12. Start 3-D printing, Get list of components along with URLs to Dr. B. for purchase + glue, fastners etc.

Part No.	Part Name & Mfg no.	Material	Qty. Rq'd	Cost per Item	Source/ Supplier	Component Weight (lb)	Weight Fraction (%)
1	Nose Cone	Fiberglass-Epoxy	1 ea.	$32 ea	Bill's Rockets bilrock.com	1.1	4.1%
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4	Forward canards	Black aluminum	4 ea.	$5.80 ea	Aerocomposites.com	2.4	9.0%
5	Wings	Black aluminum	4 ea.	$6.72 ea	Aerocomposites.com	3.8	14.2%
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total				$104.87		26.6 lb	